Inverse Stackelberg Games versus Adverse-Selection Principal-Agent Model Theory

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1 Introduction

Inverse Stackelberg games have become the subject of recent game-theory research, as a special type of Stackelberg games [2], [5]. Although at this moment only little theory about inverse Stackelberg games is available, and the theory is still in its infancy by discovering phenomena by means of examples, there seem to be many problems in various fields, which can be treated within this framework. One of the fields is the principal-agent problem treated within the economical incentive (or contract) theory [3], i.e., the problem of delegating a task to an agent, which has some private information. This situation is also known as adverse-selection.

We prove that the adverse-selection principal-agent models are a special case of inverse Stackelberg games. We find the optimal strategy of the principal maximizing his utility from the contract in both cases of contracts *with* and *without* shutdown.

This abstract is organized as follows. In Section 2 we introduce inverse Stackelberg game theory. In Section 3 we explain the basics of incentive theory restricting ourselves to an *adverse-selection case*. In Section 4 we summarize the results obtained and propose future research directions.

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2 Inverse Stackelberg Games

To introduce the concept of inverse Stackelberg games, let us consider two players, called *leader* and *follower*, respectively. Each player has its own cost function, $\mathcal{J}_L(u_L, u_F)$, $\mathcal{J}_F(u_L, u_F)$, with decision variables $u_L, u_F \in \mathbb{R}$, respectively. Each player chooses its own decision variable to minimize its own cost function. In the inverse Stackelberg equilibrium concept, the leader announces to the follower a function $\gamma_L(\cdot)$ which maps u_F into u_L. Given the function $\gamma_L(\cdot)$, the follower will make his choice u_F according to[1]:

$$u_F^* = \arg\min_{u_F} \mathcal{J}_F(\gamma_L(u_F), u_F).$$

The leader, before announcing its $\gamma_L(\cdot)$, can predict how the follower will play and he tries to choose the γ_L-function that ultimately minimizes his own cost function $\mathcal{J}_L$. Symbolically we can write:

$$\gamma_L^*(\cdot) = \arg\min_{\gamma_L(\cdot)} \mathcal{J}_L(\gamma_L(u_F(\gamma_L(\cdot))), u_F(\gamma_L(\cdot))).$$

In some cases such a function $\gamma_L^*(\cdot)$ does not exist, as we show in the paper.

3 Adverse-Selection Principal-Agent Model

Let us consider a bilateral relationship in which one party (the principal) contracts another (the agent) to delegate the production of some good [3].

The principal designs a (q, t)-contract with quantity $q \in \mathbb{N}^+$, i.e., the amount of products he

[1] Optimizing quantities will be provided with an asterisk.

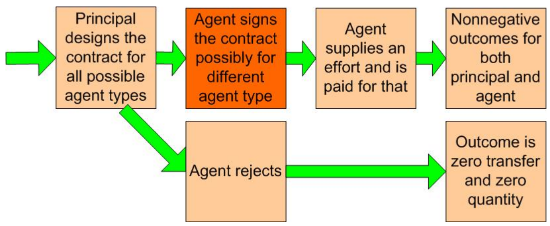

Figure 1: Adverse-selection principal-agent model.

demands from the agent, and transfer $t \in \mathbb{R}_+$, i.e., the wage he will pay to the agent for the products. It is always the principal who designs the contract, and the agent decides to sign or not to sign it.

The agent's efficiency is determined by his marginal cost $\theta \in \mathbb{R}_+$, the amount of money he has to pay to obtain one product. The principal knows only the set of the agent's possible θ, but he does not know the exact marginal cost of the agent he is facing, the principal has incomplete information. The agent's cost of obtaining q products is defined as $\mathcal{C}_A(q,\theta) = \theta\, q$, and his utility from the (q,t)-contract as $\mathcal{J}_A(q,t,\theta) = t - \mathcal{C}_A(q,\theta) = t - \theta q$. The necessary condition for him to sign is that $\mathcal{J}_A(q,t,\theta) > 0$.

The principal's value of q products is described by his objective function $\mathcal{C}_P(\cdot)$. We assume that the principal's objective function is increasing and concave with respect to $q > 0$. The principal's utility from the (q,t)-contract can be described as $\mathcal{J}_P(q,t) = \mathcal{C}_P(q) - t$. The overall schema of the principal-agent model is illustrated in Figure 1.

The principal would either contract the agent no matter how efficient the agent is (*contract without shutdown*), or the principal would like to contract only the agent with a marginal cost smaller than some certain value (*contract with shutdown*).

Generally we assume that the agent's type is from a given interval, $\theta \in [\underline{\theta}, \overline{\theta}]$, $\underline{\theta}, \overline{\theta} \in \mathbb{R}_+$, known to the principal. As a simplification we assume that the agent's type θ is from the two element set $\Theta = \{\underline{\theta}, \overline{\theta}\}$, $\overline{\theta} > \underline{\theta}$. We prove that it pays to the $\underline{\theta}$-agent to mimic the $\overline{\theta}$-agent.

One of the problems we are dealing with in the paper is to find the optimal principal's strategy under assumption that the agent is of type $\underline{\theta}$ with probability $\mu \in (0,1)$ and of type $\overline{\theta}$ with probability $1-\mu$, and that the principal offers both $(\underline{q}, \underline{t})$ and $(\overline{q}, \overline{t})-$contracts to find

$$\max_{\{(\overline{t},\overline{q}),(\underline{t},\underline{q})\}} \mu\left(\mathcal{C}_P(\underline{q}) - \underline{t}\right) + (1-\mu)\left(\mathcal{C}_P(\overline{q}) - \overline{t}\right)$$

subject to

$$\mathcal{J}_A\left(\underline{q}, \underline{t}, \underline{\theta}\right) = \underline{t} - \underline{\theta}\underline{q} > 0,$$
$$\mathcal{J}_A\left(\overline{q}, \overline{t}, \overline{\theta}\right) = \overline{t} - \overline{\theta}\overline{q} > 0.$$

If it would not pay to the agent to cheat, both inequalities

$$\underline{t} - \underline{\theta}\underline{q} \geq \overline{t} - \underline{\theta}\overline{q},$$
$$\overline{t} - \overline{\theta}\overline{q} \geq \underline{t} - \overline{\theta}\underline{q},$$

would be satisfied. We show that this is not the case and that the $\underline{\theta}$-agent will always mimic the $\overline{\theta}$-agent.

In the paper we show the optimal strategy of the principal under the assumption that the agent chooses the contract ensuring him the highest possible profit.

4 Conclusions

We present the adverse-selection principal-agent model from the economical theory of incentives as an inverse Stackelberg game. Assuming that the agent always maximizes his utility possibly by mimicking the agent of a different type, we find the optimal strategy for the principal with and without shutdown.

In the future research we will investigate the one principal - more agents model as a one leader - more followers inverse Stackelberg game.

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