NON-PROLIFERATION REGIMES, IMMORAL AND RISKY: A GAME-THEORETIC APPROACH

David Koepsell and
Kateřina Staňková

Faculty of Technology, Policy, and Management
Delft University of Technology
2600 AA Delft
The Netherlands

David Koepsell is Assistant Professor of Ethics, Delft University of Technology, Netherlands and Senior Fellow, 3TU Centre for Ethics and Technology. He is author of *Who Owns You? The Corporate Gold Rush to Patent Your Genes* (Wiley-Blackwell 2009) and *Innovation and Nanotechnology* (Bloomsbury Academic 2011). http://www.davidkoepsell.com.

Kateřina Staňková is an assistant professor at Maastricht University, Networks & Strategic Optimization Group, Department of Knowledge Engineering. Her research concentrates upon Game-Theoretical and Optimal Control Methods in Systems Biology, Evolutionary game theory, predator-prey multiple-player models, Games in control of complex systems (e.g. biological systems, traffic systems) and Stackelberg games, inverse Stackelberg games.

The dominant institutional approach to nuclear weapons (and other WMDs), has been "containment." During the Cold War, nuclear technology was kept classified in the hopes that other states would not develop nuclear weapons, while the two superpowers amassed enormous stores of weapons. In the 1970s, a non-proliferation treaty promised to cut the stores of weapons of the superpowers, and required signatory states to forego nuclear weapons development. We contend that this dominant non-proliferation regime is both immoral and risky. Its immorality is based on past-actions, fair assumptions, and autonomy. We demonstrate its riskiness with a one-shot game-theoretic model that is an extension of the models that dominated cold war nuclear weapons policies. This model shows that the risk of first-use drops dramatically as the number of players increases.

INTRODUCTION

Immediately after World War II, and due to the USA's successful development and use of the world's first nuclear weapons, the focus of the Soviet Union and the US shifted from the goal of defeating the Nazis to containing each other. While some who had worked on the development of the A-bomb suggested that the moral course was to publicize the technology and immediately offer to disarm, others embraced the inevitability of an arms race and ascribed to the goal of dominance. When, in 1949, the Soviets successfully tested their own nuclear device, the arms race was on and both sides pursued the even more devastating H-bomb. Once perfected, and with the parallel development of advanced delivery devices such as rockets and high-altitude bombers, weapons production and accumulation reached two peaks (Cirincione, 2007). The total megatonnage of yield available to the US peaked at around 20,000 in the mid 1960s, and for the USSR, peaked at about the same level in the late 1970s and early 1980s. Total numbers of weapons held by each side were in the range of 30,000 to 45,000 (Cirincione, 2007).

Throughout the 1950s and 1960s, nuclear weapons technologies remained out of reach of all but a few of the richest, most technologically advanced countries. While the US and the USSR each focused on policies designed to contain the other, the spread of nuclear technology to countries allied with them was encouraged in some cases by cooperative ventures, or perceived as an opportunity to strengthen cooperative defense treaties (such as NATO). Nuclear states were mostly, for the time being, neatly divided geopolitically as either US or USSR-allied, or at least Western bloc vs. Eastern bloc. Even so, the numbers of these states, and the weapons possessed by each, were (relatively) small (Brodie, 1978). But as knowledge of the underlying science and technology spread, and as access to the raw materials and wealth necessary to develop a nuclear weapon became more available, other non-aligned nations entered the fray. With the prospect of the nuclear monopoly held by the superpowers becoming broken by third parties who were not clearly allied with either side, and thus who were apparently unpredictable, and with the goal of stepping down the huge and expensive stores of nuclear weapons stockpiled on either side, the international Non-Proliferation Treaty was initiated in 1968 and ratified by 1970. among its provisions, now agreed to by 187 nations, is an agreement by all "non-nuclear" states to pursue only peaceful uses of nuclear energy, foregoing weapons development, and agreements by nuclear states to reduce their stockpiles (Bunn, 2003).

As knowledge of the technology spread, and as access to the raw materials and wealth necessary to develop a nuclear weapon became more available, other non-aligned nations entered the fray.

By all accounts, the non-proliferation treaty is a success and since then the world's stockpiles of nuclear weapons have shrunk, with the help of the various Strategic Arms Limitations and Reduction Treaties among the superpowers (SALT and START). And even though the nuclear club has added a few members who defied or ignored the treaty's restrictions (including South Africa, India, Pakistan, and now North Korea, and unofficially Israel), the non-proliferation model has been agreed generally to be a success (Bunn, 2003). The success of this model has been applied to other weapons of mass destruction, and there are chemical and biological weapons ban treaties. In each case, treaties prohibiting the weaponization of dual-use (technologies with both peaceful and belligerent uses) science and technology have been negotiated among the superpowers after those technologies have been weaponized. Currently, there are those who would apply this model to the emerging technologies of synthetic biology and nanotechnology before weapons have been developed. Non-proliferation, after all, seems to have worked so successfully with nuclear technologies, and despite some notable scoff-laws, most of the world is free of nuclear weapons technology today, perhaps largely due to the dominant non-proliferation regime of the past forty years (Altmann and Guburd, 2004).

We should ask, before we embark upon the strategies employed during the cold war, successful or not, whether they were a) moral on their face, and b) effective at reducing the real risk of nuclear weapons: global nuclear war. Only if these two conditions are truly met should we continue this strategy with future dual-use technologies, especially where the economic and social benefits of these technologies may be so high, their pursuit so difficult to contain, and the risks so far so few.

ETHICAL AND PRACTICAL CONSIDERATIONS

Some practical problems with containment

The policy of containment was both militarily strategic and ideologically-motivated. Arguably the result of George Kennan and President Truman, containment was aimed at not only stopping Soviet nuclear dominance, but containing the spread of communist ideology. To accomplish both aims, the US embarked on a massive technological and scientific push to create better, more deadly nuclear weapons and place them into operation with modern delivery mechanisms, all according to the game-theoretical model of *deterrence* (Kennan, 1987). The Soviets did the same. Immediately following the successful completion of the Manhattan Project, some argued that the only way to prevent an all-out arms race, and to prevent an eventual nuclear war, was to make public the knowledge behind the device, and unilaterally disarm, showing to the world the respect for the destructive power of nuclear weapons, while admitting that even while the US would not seek to monopolize the weapons, it would also not seek to stockpile or ever use them again. Of course, some argued that unilateral disarmament (or refusal to arm) was suicidal and that other nations (primarily the USSR) could not be trusted to do the same. The only effective way to stop the Soviets from building, stockpiling, and eventually using their weapons dominance to bring the rest of the world into their ideological orbit was a strategy of deterrence. By this strategy, the US would achieve dominance in the technology early, monopolize the science, and then use their nuclear threat to prevent Soviet aggression, limit their geopolitical spread, and maybe even shrink their influence.

Deterrence is based upon a standard two-player prisoner's dilemma model for cooperation if a rational actor understands the punishment or cost of defection.

Deterrence as a military strategy is based upon a standard, iterated two-player prisoner's dilemma model by which cooperation can be assured if a rational actor understands the punishment or cost of defection. (Axelrod 1984). This theory works as part of the standard assumptions in systems of criminal punishment.

But nuclear war is not like robbing a bank, and in the cold war, the strategies devised by each side rapidly became quite complex. Simply put, deterrence theory and containment policies rapidly devolved into a nuclear arms race with the appropriately-named moniker "MAD" or mutually-assured destruction, as the operating principle. Because nuclear weapons could eventually be deployed through numerous means, including submarines, hardened silos, mobile rockets, and constantly airborne bombers, each side had to ensure that it could survive a first strike to deliver a sufficiently harmful second strike. Only the threat of a deadly second strike, or mutual destruction in case one side defects, could make deterrence possible. To ensure a second strike capability in light of ever-evolving technologies, each side had to amass far more megatonnage than would ever make rational sense for a first strike. Of course, each side seeing the other's weapons buildup must also respond, assuming that the other side has a first-strike capability, and so for maximal survivability for a second-strike, maintain a constantly deadly store (Muller, 2004).

Containment and MAD did not necessarily go hand in hand, either. The moral stigma associated with the use of nuclear weapons prevented their strategic use in numerous ideologically-based conventional wars fought between the superpowers as the US sought to contain Marxist ideology. Culminating with the Cuban Missile Crisis, MAD strategy neither apparently contained ideology nor the threat of aggression, as each side came to the brink of nuclear war, backing down only through backchannel diplomacy and a secret deal to withdraw US missiles from Turkey, where they had been threatening the Soviets at as close a range as Soviet missiles threatened the US from Cuba. MAD required "brinksmanship," or the psychological stance of being perceived by one's opponent as willing not only to go to the brink, but to step over it, in order to be effective deterrence. President Nixon one-upped MAD with his own "madman" policy, consciously projecting potential mental illness as a means to intimidate potential aggressors (Perlstein, 2008).

Ultimately, building and maintaining a sufficiently large nuclear capability to ensure MAD as a deterrent proved costly for both sides, and made an arguably still-lasting impact on US national debt. With the collapse of the USSR, nuclear threats did not disappear. Even while Russia and the US have negotiated significant reductions in their stockpiles, tens of thousands of nuclear weapons remain, poised for use, although not specifically targeted at any one moment. The potential for a full-blown nuclear exchange, with unfathomable world-wide consequences, remains. The policy of containment, while no longer ideologically-based, has shifted. Non-proliferation policies now attempt to steer development of nuclear science toward peaceful uses, restricting the spread of both enrichment and delivery mechanisms to an elite club, the nuclear "haves" and excluding permanently the nuclear "have-nots" (Litwak, 2000). Because nuclear technologies are difficult and expensive to develop, containment by non-proliferation treaties and institutions has been more-or-less possible, with notable exceptions, though its future is perhaps in doubt. But is it moral?

Some moral problems with containment

Containment, at least as practiced historically and extended through the current non-proliferation regime, offers a number of moral challenges both actual and hypothetical. The US has thus far been the only state to have used nuclear weapons in warfare. Having done so, albeit in the context of world war, it then insisted upon monopolizing the technology. Even after nuclear technology spread, the science remained classified, nuclear materials remained outlawed for all except a select club of those who were politically-allied and often restricted to peaceful uses, even while the US increased its stockpiles of weapons. The lack of symmetry seems unfair, and is arguably unethical (Lee, 1985). Who has a right to which science and what technologies? Who is to mediate that right? Ought a state that has so far monopolized such weapons' use be the arbiter of whom else should posses such a capability?

Containment as a policy assumes that the currently-possessing states are morally entitled to nuclear weapons while others are not. Negotiating with non-possessing states cannot be fair either as possessing states are so far unwilling to renounce their own weapons, and so effectively blackmail non-nuclear states into agreeing not to build weapons in exchange for economic benefits, and transfer of peaceful nuclear know-how. This is hardly a fair position from which to bargain, and the bulk of those signing the non-proliferation treaty had no leverage to agree to anything else (Granoff, 2000). Moreover, from a Rawlsian perspective we ought not to choose such asymmetry. Imagining states as we would actors forming states, standing behind the "veil of ignorance," we cannot be assured that we will be, among the panoply of potential states, a nuclear state. Rather, we should assume as Rawls encourages that we will be the "least" among possible actors, and thus be a non-nuclear state, constrained by the threat of nuclear "containment" strategies, and existing treaties signed under the assymetrical threat posed by the few nuclear states. From the Rawlsian perspective of justice as fairness, and employing his model, we ought to not

Containment as a policy assumes that the currently-possessing states are morally entitled to nuclear weapons while others are not.

tolerate a system that exacerbates inequalities (Rawls 1971). After all, if we envision a *just* world from behind Rawls' "veil of ignorance," we cannot assume that we will emerge as a nuclear state.

Who has the right to curtail the spread of either basic scientific knowledge or technology of any kind? If it is a proper matter for states, then bargains should be freely entered into, and those having the strength of first-mover status ought to be prepared to enter into a fair bargaining position. The current non-proliferation regime is inherently unfair because of its striking lack of symmetry. It does not require nuclear states to become non-nuclear states. Rather, it only encouraged overall reductions in stockpiles held by nuclear states. The haves agreed in principle only on the abstract notion that it would be nice perhaps someday if there were no nuclear weapons at all, and to ensure this, non-nuclear states were pressured into agreeing to prohibition, while the nuclear states would be trusted to keep their weapons, reduce them a bit, and police the world as part of a still-privileged club.

There is no supportable ethical claim to any intellectual property involved. Monopolizing fundamental scientific knowledge is itself unethical. The current system, which forces states to agree not to pursue what would otherwise be regarded to be part of the scientific commons defies the principles, or what Robert Merton called the *ethos* of science, which demands universalism, communalism, disinterested and organized skepticism (Merton, 1973). Containment policies and non-proliferation regimes unethically prevent the use of fundamental science based upon political agreements. The scientific commons encompasses all laws of nature and their applications, and attempts to monopolize either are arguably unethical breaches of the freedoms of conscience and expression (Koepsell 2011).

There is no supportable ethical claim to any intellectual property involved. Monopolizing fundamental scientific knowledge is itself unethical.

Finally, non-proliferation regimes do little to deter criminals from trying to possess WMD, and criminals are not signators to treaties. While treaties make states responsible for curtailing the use of nuclear and other technologies for weapons, rogue groups, terrorists, and criminals are free to attempt to possess, profit from, and use these technologies outside of international law, checked only by legal mechanisms and police actions by capable states (Litwak, 2000). With new technologies such as nanotechnology and synthetic biology, police or intelligence operations that attempt to contain dual-use technologies seem doomed to failure. Not only may states be prevented from pursuing weaponization of these technologies, but they may be powerless to prevent criminals from doing so, making themselves susceptible to attacks both from within and without, prevented meanwhile from the scientific or technical know-how needed to stop criminals, defend themselves, or retaliate.

Of course, the most moral path is to *choose* not to develop nuclear weapons in the first place, or having done so, to unilaterally agree to disarm. At least, unilateral drawdown and converting from a state of nuclear readiness would be preferable morally before requiring the world to agree to not developing similar weapons and stockpiling them (Tannenwald, 1999). In the post-cold war environment, the WMD-possessing superpowers will gain the moral high ground, and thus moral credibility in the argument for disarmament by others, by radical drawdown or complete disarmament, as well as openness about the underlying science and technology involved in both classical and advanced WMD. The alternative, and what has apparently been chosen, is the use of implicit threat if not force. By maintaining monopolies over the science and technology, imposed now also through international institutions, the moral argument for disarmament has been lost (Goodin, 1985; Sagan, 1996).

Some may argue that what has been gained is increased security. Weapons of mass destruction are simply too dangerous to be open to all, and in this case, a legally and militarily-enforced monopoly is necessary to assure the safety of all, even at the expense of moral, democratic principles. After all, the cold war stalemate worked, the peace was kept, and nuclear weapons were largely contained within the existing superpowers (with minor exceptions). The same formula should work to our mutual safety in the post cold-war environment and with emerging WMD technologies.

We contend that these assumptions are not necessarily true. We may have simply been lucky to survive the cold war, and in a multi-polar world, in which two superpowers no longer dominate economically or geopolitically, our assumptions about non-proliferation and its benefits should be reconsidered. Below, we apply game theory to show why non-proliferation is not only immoral, but risky as a strategy for keeping the peace, and argue that a new approach must be taken. The game theoretical approach, used as the primary tool to devise cold war nuclear strategy, demonstrates that the potential for actual first use of nuclear weapons (or other WMD, presumably) *goes down* with an increasing number of states possessing such weapons. This would seem to argue for *proliferation* (though not necessarily stockpiling) as a moral and *safer* alternative to monopolization.

THE GAME

Motivation

We will model the interaction among the countries possessing nuclear weapons using game theory. Standard literature dealing with arms race models (Wallace, 1978; Richardson, 1960) focuses on the dynamic process of the arms race over the time or on fitting real data collected over the years using this model. Our aim is different: we wish to see what is the probability that nuclear weapons are used once the weapons are possessed by more countries; therefore the problem is defined as a one-shot (static) game (Başar, 1999). Game theory is a useful tool to model interactions of the parties with conflicting interests. Unlike in the classical literature dealing with nuclear war using game theory (see (O'Neill, 1994) and references therein), we propose a model with continuous decision spaces, i.e., the countries (as players) do not decide whether to attack or not, but they decide about which portion of the weapons that they possess they should use. Models with continuous decision spaces prove themselves more realistic and are used also in various fields (Başar, 1999; Staňková 2009). Such models cannot be transformed into matrix games anymore and are in general more complex to deal with; however, they provide more realistic insight into the problems.

In this paper we propose a model with continuous decision spaces, i.e., the countries (as players) do not decide whether to attack or not, but they decide about which portion of the weapons that they possess they should use.

Basics (full game is in the Appendix)

In general, a (one-shot) game is defined by the set players, their decision sets, and outcomes for each of the players for each of the combination of decisions of all players (Başar, 1999).

We consider an *N*-person game between $2 \leq N$ countries as players (we will use the term "countries" to refer to the authorities of individual countries) that may possess a nuclear weapon. We consider a situation in which a country decides whether to attack or not. In mathematical terms, we can define a decision $u_i \in [0,1]$ for country *i*, where $u_i = 0$ represents "to not attack" and $u_i = 1$ represents "to attack," $u_i \in (0,1)$ a mixed (peaceful/attack) decision. The decision u_i can be then interpreted as a probability that *i* will attack or as the portion of the weapons that *i* will use. Without the loss of generality, in the remainder of this section we will confine ourselves to the latter interpretation.

We assume that each of the countries has utility from using its nuclear weapons. The attitude of each country towards the risk defines the form of its utility function. Each of the countries can be either risk-affine (risk-seeking, utility function convex), risk-neutral (utility function linear), or risk-averse (utility function concave) (Laffont, 2002). Moreover, while a country has no utility from not attacking, its utility should monotonically increase with the amount of weapons used, which also means that it increases with u_i (if n_i represents the total amount of the nuclear weapons available to country *i*, $u_i \cdot n_i$ represents the amount of weapons used). Fig. 1 in the Appendix depicts the possible forms of utility functions. However, after attacking, the country is punished by other countries possessing nuclear weapons. The countries not possessing nuclear weapon are not included into the model because the strength of their possible punishment is assumed to be much lower than the strength of the punishment by countries with nuclear weapons.

The upshot of the game is actually intuitive, although it counters accepted nuclear deterrence strategy and theory to date. Essentially, we assume that the cost of using nuclear weapons in a first strike goes up in direct relation to the opportunity for retaliation by the world community. With more nuclear states, the greater the costs via punishment by other nuclear states. As well, the value of a first strike falls with greater proliferation such that, at a certain point, the strategy of a first strike becomes unworkable. The policy implcations, considered in conjunction with the ethical arguments, suggest that non-proliferation regimes as we continue to argue for and enforce them, are both unethical and risky. They increase the value and reduce the potential costs of a first strike. Of course, proliferation still carries the risk of accidental use, or use by criminals of nuclear technologies, but if the highest goal is to prevent use of nuclear weapons by states in a nuclear exchange, or to prevent all-out nuclear war, then the game we posit suggests that proliferation will better achieve this.

CONCLUSIONS

Using a one-shot game-theoretic model of nuclear war we have shown that if more players are involved in the game, it is less probable that any of the countries will use their nuclear weapons in a first strike. We have provided a lower bound on the number of players in the game (in terms of other parameters) which ensures that the Nash equilibrium coincides with the situation in which no country attacks the others. Although the proposed model is rather simple, we believe that it depicts most essential components of the utilities of the possible attackers. The model can be extended in many ways, however, we believe that its current version illustrates our point clearly enough.

Using a one-shot game-theoretic model of nuclear war we have shown that if more players are involved in the game, it is less probable that any of the countries will use their nuclear weapons in a first strike.

Some will object that this argument also suggests that individual or personal armament is the best way to deter crime, a position held by some pro-gun groups in the US and elsewhere. This is not necessarily implied by our model as the motivations for crime are presumably much different than those for acts of warfare. Nor are open science, technology, and possessory rights a panacea to eliminating nuclear or other WMD violence. Rogue states, criminals, and terrorists will doubtless attempt to and eventually possess WMD. But the current non-proliferation regime, especially in light of emerging, cheap, and difficult to detect technologies, will likely do little to prevent this possibility. What openness allows is for more good-intentioned actors to know about the technologies and hopefully help stop bad-intentioned actors before it is too late.

Finally, recognizing the immorality of the current non-proliferation regime, and the risk of monopolization of WMD technologies in just a few hands, does not imply that seeking or possessing these weapons is itself moral. On the contrary, the most moral approach is to forego such weapons at the start, to jointly agree to peaceful uses of dual-use technologies and sciences, and to avoid belligerent intentions or use+s. We hope that in a multi-polar, freely proliferating world, the rational choice will become obvious, stockpiles will lose their value as the cost of first use climbs, and nations will be compelled by a new climate to seek multilateral disarmament, propelled not by threat, but rather impelled by the best intentions.

Bibliography

Altmann, Jürgen and Gubrud, Mark A., 'Military, Arms Control, and Security Aspects of Nanotechnology' in *Discovering the Nanoscale*, D. Baird, A. Nordmann & J. Schummer (eds.), IOS Press, Amsterdam.

Axelrod, R. (1984). *The Evolution of Cooperation.* ISBN 0-465-02121-2

Başar, Tamer and Olsder, Geert J., 1999. *Dynamic Noncooperative Game Theory.* SIAM, Philadelphia.

Brodie, Bernard, 1978. 'The Development of Nuclear Strategy,' *International Security*, Vol. 2, No. 4, pp. 65-83.

Cirincione, Joseph, 2007. *Bomb Scare: The History and Future of Nuclear Weapons.* Colubia University Press, New York.

Goodin, Robert E., 1985. 'Nuclear Disarmament as a Moral Certainty,' *Ethics*, Vol. 95, No. 3, Special Issue: Symposium on Ethics and Nuclear Deterrence April, pp. 641-658.

Granoff, Jonathan, 2000. 'Nuclear Weapons, Ethics, Morals, and Law' *Brigham Young Law Review*, Vol. 2000: pp. 1413-1432.

Kennan, George, F., 1987. 'Containment, Then and Now' in *Foreign Affairs*, Vol. 65, No. 4, Spring, pp. 885-890.

Koepsell, David, 2011. *Innovation and Nanotechnology: Converging Technologies and the End of Intellectual Property.* Bloomsbury Academic, London.

Laffont, Jean-Jacques and Martimort, David, 2001. The Theory of Incentives: The Principal-Agent Model. Princeton University Press, Princeton.

Lee, Steven, 1985. 'The Morality of Nuclear Deterrence: Hostage Holding and Consequences' *Ethics*, Vol. 95, No. 3, Special Issue: Symposium on Ethics and Nuclear Deterrence April, pp. 549-566.

Litwak, Robert S., 2000. *Rogue States and U.S. Foreign Policy: Containment After the Cold War*, The Johns Hopkins University Press, Maryland.

Merton, Robert K. (1973), "The Normative Structure of Science", in Merton, Robert K., *The Sociology of Science: Theoretical and Empirical Investigations*, Chicago: University of Chicago Press

Muller, Richard R. 2004. 'The Origins of MAD: A Short History of City-Busting' in *Getting MAD: Nuclear Mutual Assured Destruction, Its Origins and Practice*, Henry D. Sokolski (ed.) US Army War College Press.

O'Neill, Barry, 1994. 'Game theory models of peace and war,' In *Handbook of Game Theory with Economic Applications*, Ed. 1, Vol. 2, Chap. 29, pp. 995-1053, Elsevier.

Perlstein, Rick, 2008. *Nixonland: The Rise of a President and the Fracturing of America*, Scribner, New York.

Rawls, John, 1971 *A Theory of Justice*, The Belknap Press of Harvard University Press, Cambridge, USA.

Richardson, Lewis F., 1960. *Arms and Insecurity.* Quadrangle Press, Chicago.

Sagan, Scott D., 1996. 'Why Do States Build Nuclear Weapons?: Three Models in Search of a Bomb', *International Security*, Vol. 21, No. 3, Winter, pp. 54-86.

Staňková, Kateřina., Olsder, Geert J. and Bliemer, Michiel C.J., 2009. 'Comparison of Different Toll Policies in the Dynamic Second-best Optimal Toll Design Problem: Case study on a Three-link network,' *European Journal of Transport and Infrastructure Research*, Vol. 9, No. 4, pp. 331-346.

Staňková, Kateřina, Olsder, Geert J., and De Schutter, Bart, 2010. 'On European electricity market liberalization: A game-theoretic approach,' *INFOR: Information Systems and Operational Research*, Vol. 48, No. 4, pp. 267-280.

Tannenwald, Nina, 1999. 'The Nuclear Taboo: The United States and the Normative Basis of Nuclear Non-Use,' *International Organization*, Vol. 53, No. 3, Summer, pp. 433-468.

Wallace, Michael D. and Wilson, Judy M, 1978. 'Non-Linear Arms Race Models,' *Journal of Peace Research*, Vol. 15, No. 2, pp. 175-192.

APPENDIX, THE GAME

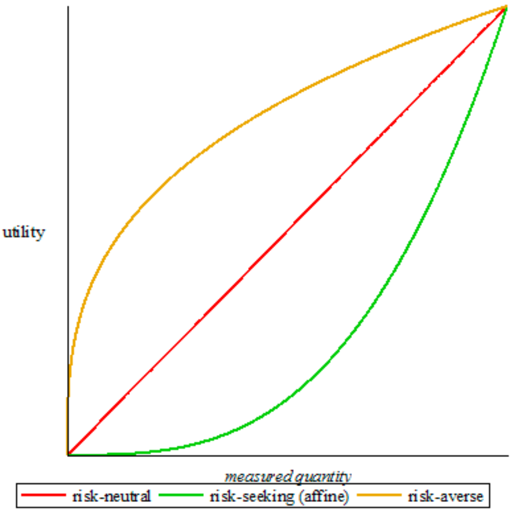


Fig.1: Utility functions

Country i maximizes profit L_i, which can be defined as

$$L_i(\hat{u}_1,\dots,\hat{u}_{i-1},\ \hat{u}_{i+1},\dots,u_N) = P_i^A(u_i,n_i) - C_i^D(u_i,n_i) - C_i^A(\hat{u}_1,\dots,\hat{u}_{i-1},\ \hat{u}_{i+1},\dots,u_N) \tag{1}$$

Here the first term, $P_i^A(u_i,n_i)$, represents a profit of country i from attacking, monotonically increasing with u_i, the quantity of the weapons that country i possesses, n_i, and with the number of weapons that are used, $u_i \cdot n_i$. Note that n_i is the state variable, i.e., it is not assumed that country i decides (in this game) about how many weapons it produces. Such an assumption can be made if we expect that the decision about the amount of nuclear weapons produced was taken a priori.

$C_i^D(u_i,n_i)$ represents the costs to develop and use the weapons, monotonically increasing with u_i, the quantity of the weapons that country i possesses, n_i, and therefore also with the number of weapons that are used, $u_i \cdot n_i$.

$C_i^A(\hat{u}_1,\ldots,\hat{u}_{i-1},\ \hat{u}_{i+1},\ldots,u_N)$ represents the cost that country *i* expects to pay for attacking ("punishment"). This cost is monotonically increasing with each $u_j \in (0,1]$ $(j \in N \setminus i)$, which defines the expected strength of the punishment by country *j*.

The expected decision $\hat{u}_j$ of player *j* is assumed to maximize certain ("punishment") criteria $\hat{L}_j(u_i,\hat{u}_j)$ of this player. Country *i* does not know of such criteria.

Assuming differentiability of all three elements of (1) and risk aversion of the countries (implying concavity of $L_i(\hat{u}_1,\ldots,\hat{u}_{i-1},\ \hat{u}_{i+1},\ldots,u_N)$, if $u_i=0$ is the best choice for country *i* then

$$\frac{\partial L_i}{\partial u_i}=\frac{\partial P_i^A(u_i,n_i)}{\partial u_i}-\frac{\partial C_i^D(u_i,n_i)}{\partial u_i}-\frac{\partial C_i^A(\hat{u}_1,\ldots,\hat{u}_{i=1},u_i,\hat{u}_{i+1},\ldots,\hat{u}_N)}{\partial u_i}<0 \tag{2}$$

for each $u_i \in [0,1]$

In other words, the decrease of the net profit from attacking with respect to u_i has to be higher than decrease of the punishment with respect to the same quantity:

$$\frac{\partial P_i^A(u_i,n_i)}{\partial u_i}<\frac{\partial C_i^D(u_i,n_i)}{\partial u_i}+\frac{\partial C_i^A(\hat{u}_1,\ldots,\hat{u}_{i=1},u_i,\hat{u}_{i+1},\ldots,\hat{u}_N)}{\partial u_i} \tag{3}$$

We argue that while the left-hand side of (3) increases with the force of the attack (number of weapons used) and it and does not generally increase with more countries involved in the game.

The right-hand side of (3), increases with number of the players involved.

To illustrate our reasoning, we show analysis of the game with more specific choice of $P_i^A(u_i,n_i)$, $C_i^D(u_i,n_i)$, and $C_i^A(\hat{u}_1,\ldots,\hat{u}_{i-1},\ \hat{u}_{i+1},\ldots,u_N)$. The following assumption is imposed just for the sake of the simplicity of the analysis and does not affect the general outcome of this study.

Assumption 1

Player i does not distinguish between other players. It does not know n_j for $j \in N \setminus \{i\}$ and assumes that each country $j \in N \setminus \{i\}$ has the same expected $\hat{n}_j$. Similarly, u_j is the same for all $j \in N \setminus \{i\}$.

More specific choice of the profit function for the *i*-th player: risk averse players

If country i is risk averse, $P_i^A(\cdot, n_i)$ is a concave function. Moreover, it has to satisfy $P_i^A(0, n_i) = 0$. In accordance with economic literature (Laffont, 2002) we define P_i^A as a specific function of this type:

$$P_i^A = \alpha_i \ln\left((u_i+1)n_i\right). \tag{4}$$

Here $\alpha_i > 0$ determines the impact factor of the attack. See Fig. 2 for the plot of the function P_i^A for different values of α_i And u_i with n_i normalized to 1.

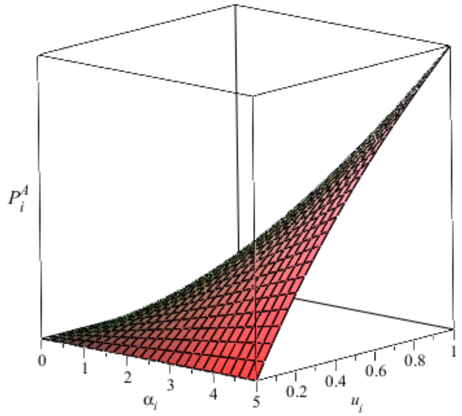


Fig. 2 Function $P_i^A = \alpha_i \ln(u_i + 1)$

Function $C_i^D(\cdot, n_i)$ (costs to develop and use the weapons) is a strictly increasing function defined on R_+^0. Moreover, it has to satisfy $C_i^D(0, n_i)=0$.

We will set this function to $C_i^D(u_i, n_i)=\beta_i \cdot u_i \cdot n_i$, (5)

tacitly assuming that the relationship between the number of weapons produced and their cost is linear (although any strictly increasing function increasing with β_i, satisfying $C_i^D(0, n_i)=0$, $C_i^D(u_i, 0)=0$, would be sufficient). Here β_i determines the importance factor of the attack costs (for *i*).

See Fig. 3 for the plot of the function C_i^D for different values of β_i With n_i normalized to 1.

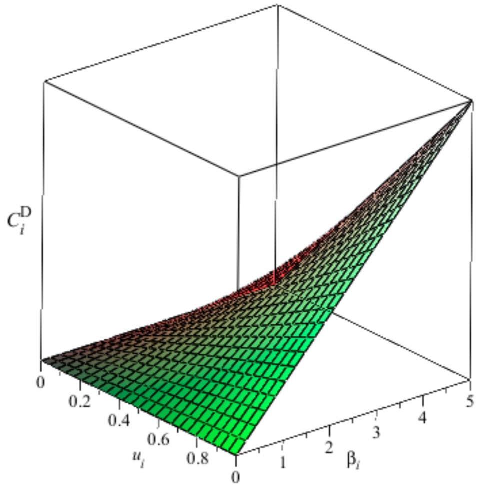


Fig. 3 Function $C_i^D(u_i, n_i)=\beta_i \cdot u_i \cdot n_i$ with n_i normalized to 1.

The cost that country *i* expects to pay for attacking ("punishment") is in the view of this country equivalent to the negative of the sum of the profits of all other countries with nuclear weapons from this punishment. As we assume risk aversion of each player $j \in N \setminus \{i\}$ and that the cost has to be equal to zero if no attack takes place and increasing with the attack strength, such cost can be then defined as

$$C_i^A(\hat{u}_1,\ldots,\hat{u}_{i=1},u_i,\hat{u}_{i+1},\ldots,\hat{u}_N) = \sum_{j\in N\setminus\{i\}} \hat{\gamma}_j \ln\left(\hat{u}_j+1\right)\hat{n}_j\left(u_i+1\right) \tag{6}$$

with $\hat{\gamma}_j$ being the expected impact factor of the punishment from country *j*. If Assumption 1 holds, this expected impact factor is the same for each $j \neq i$, i.e., $\hat{\gamma}_j = \hat{\gamma}$ for each $j \in N \setminus \{i\}$. Obviously, (6) could be replaced by a different function of similar properties and if Assumption 1 did not hold, $\hat{\gamma}$ could be set as a maximum, minimum, or an average over all $\hat{\gamma}_j$, depending on the perception of player *i*.

See Fig. 4 for the plot of the function C_i^A for a two-player game (N={1,2}), $\hat{n}_j = 1$, $\hat{\gamma}_j = 1$.

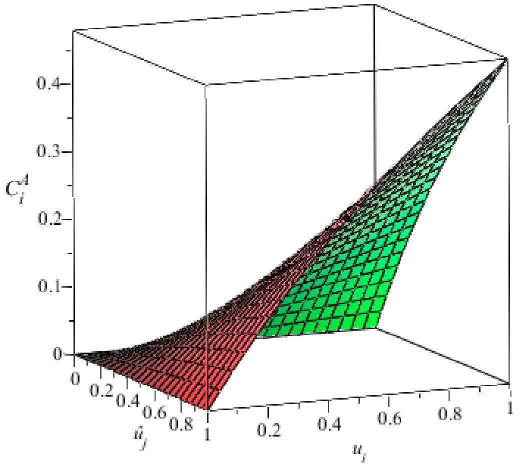


Fig. 4 Function C_i^A **for a two-player game** (N={1,2}), $\hat{n}_j = 1$, $\hat{\gamma}_j = 1$.

Substituting (4)-(6) into (1) leads to

$$L_i = \alpha_i \ln((\mathrm{u_i}+1)\mathrm{n_i}) - \beta_i \mathrm{u_i n_i} - \hat{\gamma}\ln(\mathrm{u_i}+1)\ln(\mathrm{u_j}+1)(\mathrm{N}-1)\mathrm{n_j} \tag{7}$$

Then

$$\frac{\partial L_i}{\partial u_i}=\frac{\alpha_i-\beta_i n_i(u_i+1)-(N-1)\hat{\gamma}\ln(u_j+1)n_j}{(u_i+1)}=-\beta_i n_i+\frac{\alpha_i-(N-1)\hat{\gamma}\ln(u_j+1)n_j}{(u_i+1)} \tag{8}$$

L_i decreases with u_i if (8) is negative. In such a case the optimal choice for the player i would be $u_i = 0$, i.e., no attack would take place.

Expression (8) is negative if

$$N>\frac{\alpha_i-\beta_i n_i(u_i+1)+\hat{\gamma}\ln(u_j+1)n_j}{\hat{\gamma}\ln(u_j+1)n_j} \tag{9}$$

for each $u_i \in [0,1]$. Because the right hand side of (9) decreases with u_i and (9) has to be satisfied for each $u_i \in [0,1]$, which implies

$$N>\frac{\alpha_i-2\beta_i n_i+\hat{\gamma}\ln(u_j+1)n_j}{\hat{\gamma}\ln(u_j+1)n_j} \tag{10}$$

Please note that the left-side of equation (10) would become infinite if the expected punishment $u_j = 0$. In such a case (10) would not be satisfied for any N. However, as weassume that u_j is nonzero, such a situation cannot happen. Higher u_j is, lower N has to be to satisfy (10).

Denoting the expression $\gamma\ln(u_j+1)\,n_j$ by p (punishment), equation (10) can be with nonzero p rewritten as

$$\frac{\alpha_i-2\beta_i n_i}{p}+1\leq N\ , \tag{11}$$

Where maximal p equal to $\gamma \ln 2n_j$.

If (11) is not satisfied, the probability that player i attacks (portion of its weapons to be used) is nonzero.

Lemma 1 ***(Nash equilibrium)***

If (11) is satisfied for each player $i \in N$, the Nash equilibrium of the game with profits defined by (1) is $(u_1,\dots u_N) = (0,\dots 0)$.

Lemma 1 gives a sufficient condition for the Nash game with the profits defined by (1) for each $i \in N$ to not use their weapons. In Fig. 5 you can see what is the minimal number of players assuring (1) with $n_i = n_j = 1$, $\alpha_i = 1$, $\beta_i = \frac{\alpha_i}{10}$, with respect to γ and u_j.

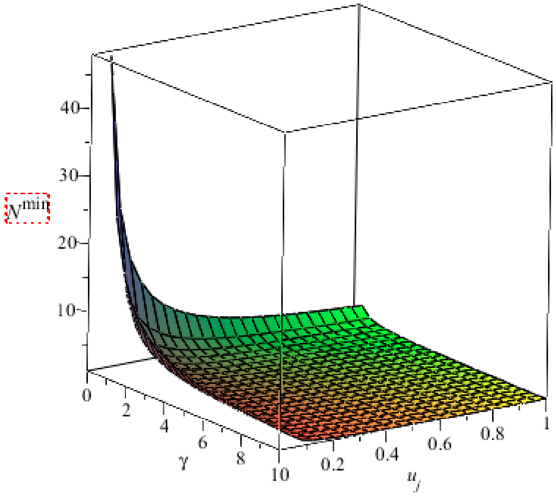
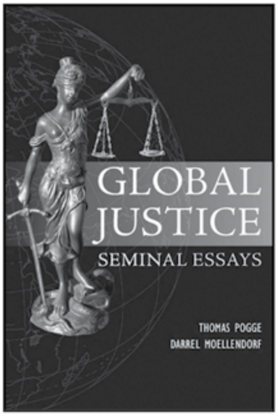
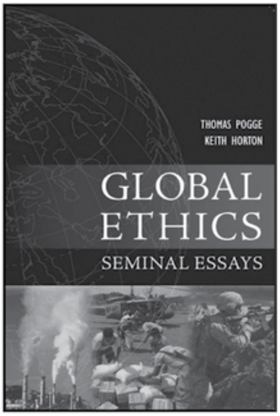


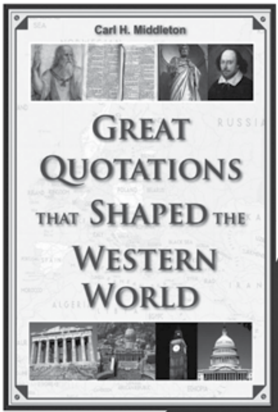
Fig.5 Minimal number of countries involved in order to assure peace, with respect to to γ and u_j ($n_i = n_j = 1$, $\alpha_i = 1$, $\beta_i = \frac{\alpha_i}{10}$).

Similar analysis can be carried out for a more general utility function as well as for risk-neutral and risk-seeking players. In all these cases, however, it is assumed that all involved parties are rational. Irrationality of any of the parties involved might lead to different results.

A very important observation is that equation (3) is more likely to be satisfied if N increases; this statement is independent of what type of functions representing the utilities of the players we choose, as long as they are continuous and increasing with the decision variable of the player in question and if they reach 0 when no attack takes place.


GLOBAL JUSTICE
SEMINAL ESSAYS
THOMAS POGGE
DARREL MOELLENDORF


THOMAS POGGE
KEITH HORTON
GLOBAL ETHICS
SEMINAL ESSAYS


Carl H. Middleton
GREAT QUOTATIONS THAT SHAPED THE WESTERN WORLD