Lecture 17: Examples in hypothesis testing

Kateřina Staňková

Statistics (MAT1003)

May 22, 2012

Outline

1. Revision of basics
2. Revision of two-sided tests vs. one-sided tests
3. Many exercises

book: Chapter 10

And now . . .

1. Revision of basics
2. Revision of two-sided tests vs. one-sided tests
3. Many exercises

Outcome of the hypothesis testing

	H_0 is true	H_0 is false
Do not reject H_0	Correct decision	Type II Error (with prob. β)
Reject H_0	Type 1 error (with prob. α)	Correct decision

Outcome of the hypothesis testing

	H_0 is true	H_0 is false
Do not reject H_0	Correct decision	Type II Error (with prob. β)
Reject H_0	Type 1 error (with prob. α)	Correct decision

P-value

The highest α such that H_0 is accepted and the lowest α such that H_0 is rejected

And now . . .

1 Revision of basics

2 Revision of two-sided tests vs. one-sided tests

3 Many exercises

One-sided vs. two-sided tests

One-sided vs. two-sided tests

So far: $H_0 : \mu = x$ vs. $H_1 : \mu \neq x$ (so-called two-sided tests)

One-sided vs. two-sided tests

So far: $H_0 : \mu = x$ vs. $H_1 : \mu \neq x$ (so-called two-sided tests)
Now: $H_0 : \mu = x$ vs. $H_1 : \mu > x$ or $H_1 : \mu < x$ (so-called one-sided tests)

One-sided vs. two-sided tests

So far: $H_0 : \mu = x$ vs. $H_1 : \mu \neq x$ (so-called two-sided tests)
Now: $H_0 : \mu = x$ vs. $H_1 : \mu > x$ or $H_1 : \mu < x$ (so-called one-sided tests)
Same as $H_0 : \mu \geq x$ vs. $H_1 : \mu < x$, or $H_0 : \mu \leq x$ vs. $H_1 : \mu > x$

One-sided vs. two-sided tests

So far: $H_0 : \mu = x$ vs. $H_1 : \mu \neq x$ (so-called two-sided tests)
Now: $H_0 : \mu = x$ vs. $H_1 : \mu > x$ or $H_1 : \mu < x$ (so-called one-sided tests)
Same as $H_0 : \mu \geq x$ vs. $H_1 : \mu < x$, or $H_0 : \mu \leq x$ vs. $H_1 : \mu > x$
We use one-sided tests for calculation purposes: Given that $\mu = x$ we use CLT and/or probability distributions

One-sided vs. two-sided tests

So far: $H_0 : \mu = x$ vs. $H_1 : \mu \neq x$ (so-called two-sided tests)
Now: $H_0 : \mu = x$ vs. $H_1 : \mu > x$ or $H_1 : \mu < x$ (so-called one-sided tests)
Same as $H_0 : \mu \geq x$ vs. $H_1 : \mu < x$, or $H_0 : \mu \leq x$ vs. $H_1 : \mu > x$
We use one-sided tests for calculation purposes: Given that $\mu = x$ we use CLT and/or probability distributions

- $H_o : \mu \geq x$

One-sided vs. two-sided tests

So far: $H_0 : \mu = x$ vs. $H_1 : \mu \neq x$ (so-called two-sided tests)
Now: $H_0 : \mu = x$ vs. $H_1 : \mu > x$ or $H_1 : \mu < x$ (so-called one-sided tests)
Same as $H_0 : \mu \geq x$ vs. $H_1 : \mu < x$, or $H_0 : \mu \leq x$ vs. $H_1 : \mu > x$
We use one-sided tests for calculation purposes: Given that $\mu = x$ we use CLT and/or probability distributions

- $H_o : \mu \geq x$
- Acceptance region: $\bar{X} \geq y$

One-sided vs. two-sided tests

So far: $H_0 : \mu = x$ vs. $H_1 : \mu \neq x$ (so-called two-sided tests)
Now: $H_0 : \mu = x$ vs. $H_1 : \mu > x$ or $H_1 : \mu < x$ (so-called one-sided tests)
Same as $H_0 : \mu \geq x$ vs. $H_1 : \mu < x$, or $H_0 : \mu \leq x$ vs. $H_1 : \mu > x$
We use one-sided tests for calculation purposes: Given that $\mu = x$ we use CLT and/or probability distributions

- $H_o : \mu \geq x$
- Acceptance region: $\bar{X} \geq y$
- Significance level: $\alpha = P(\overline{X} \leq y)$

One-sided vs. two-sided tests

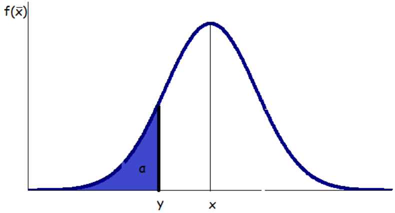
So far: $H_0 : \mu = x$ vs. $H_1 : \mu \neq x$ (so-called two-sided tests)
Now: $H_0 : \mu = x$ vs. $H_1 : \mu > x$ or $H_1 : \mu < x$ (so-called one-sided tests)
Same as $H_0 : \mu \geq x$ vs. $H_1 : \mu < x$, or $H_0 : \mu \leq x$ vs. $H_1 : \mu > x$
We use one-sided tests for calculation purposes: Given that $\mu = x$ we use CLT and/or probability distributions

- $H_o : \mu \geq x$
- Acceptance region: $\bar{X} \geq y$
- Significance level: $\alpha = P(\overline{X} \leq y)$
- P-value also one-sided

One-sided vs. two-sided tests

So far: $H_0 : \mu = x$ vs. $H_1 : \mu \neq x$ (so-called two-sided tests)
Now: $H_0 : \mu = x$ vs. $H_1 : \mu > x$ or $H_1 : \mu < x$ (so-called one-sided tests)
Same as $H_0 : \mu \geq x$ vs. $H_1 : \mu < x$, or $H_0 : \mu \leq x$ vs. $H_1 : \mu > x$
We use one-sided tests for calculation purposes: Given that $\mu = x$ we use CLT and/or probability distributions

- $H_o : \mu \geq x$
- Acceptance region: $\bar{X} \geq y$
- Significance level: $\alpha = P(\overline{X} \leq y)$
- P-value also one-sided



And now . . .

1. Revision of basics
2. Revision of two-sided tests vs. one-sided tests
3. Many exercises

Example 2 (a) – from last lecture

Example 2 (a) – from last lecture

$X_1, \ldots, X_5$ IID, $\sim \mathrm{Exp}(\lambda)$, λ unknown

Example 2 (a) – from last lecture

$X_1, \ldots, X_5$ IID, $\sim \mathrm{Exp}(\lambda)$, λ unknown
$H_0 : \lambda = 2$ vs. $H_1 : \lambda > 2$

Example 2 (a) – from last lecture

$X_1, \ldots, X_5$ IID, $\sim \mathrm{Exp}(\lambda)$, λ unknown
$H_0 : \lambda = 2$ vs. $H_1 : \lambda > 2$
(So under $H_0 : \mu = E(X_i) = \frac{1}{2}$)

Example 2 (a) – from last lecture

$X_1, \ldots, X_5$ IID, $\sim \mathrm{Exp}(\lambda)$, λ unknown
$H_0 : \lambda = 2$ vs. $H_1 : \lambda > 2$
(So under $H_0 : \mu = E(X_i) = \frac{1}{2}$)
Accept H_0 if $\bar{X} > 0.45$; calculate α

Example 2 (a) – from last lecture

$X_1, \ldots, X_5$ IID, $\sim \mathrm{Exp}(\lambda)$, λ unknown
$H_0 : \lambda = 2$ vs. $H_1 : \lambda > 2$
(So under $H_0 : \mu = E(X_i) = \frac{1}{2}$)
Accept H_0 if $\bar{X} > 0.45$; calculate α

- $\bar{X} > 0.45 \Leftrightarrow \sum_{i=1}^{5} X_i > 5 \cdot 0.45 = 2.25$

Example 2 (a) – from last lecture

$X_1, \ldots, X_5$ IID, $\sim \mathrm{Exp}(\lambda)$, λ unknown
$H_0 : \lambda = 2$ vs. $H_1 : \lambda > 2$
(So under $H_0 : \mu = E(X_i) = \frac{1}{2}$)
Accept H_0 if $\bar{X} > 0.45$; calculate α

- $\bar{X} > 0.45 \Leftrightarrow \sum_{i=1}^{5} X_i > 5 \cdot 0.45 = 2.25$
  Let $Y = \sum_{i=1}^{5} X_i$ (sum of independent exponentially distributed RV's)

Example 2 (a) – from last lecture

$X_1, \ldots, X_5$ IID, $\sim \mathrm{Exp}(\lambda)$, λ unknown
$H_0 : \lambda = 2$ vs. $H_1 : \lambda > 2$
(So under $H_0 : \mu = E(X_i) = \frac{1}{2}$)
Accept H_0 if $\bar{X} > 0.45$; calculate α

- $\bar{X} > 0.45 \Leftrightarrow \sum_{i=1}^{5} X_i > 5 \cdot 0.45 = 2.25$
  Let $Y = \sum_{i=1}^{5} X_i$ (sum of independent exponentially distributed RV's)

$\Rightarrow$ $Y \sim \mathrm{Erl}(\lambda = 0, n = 5)$ - under H_0 :

Example 2 (a) – from last lecture

$X_1, \ldots, X_5$ IID, $\sim \mathrm{Exp}(\lambda)$, λ unknown
$H_0 : \lambda = 2$ vs. $H_1 : \lambda > 2$
(So under $H_0 : \mu = E(X_i) = \frac{1}{2}$)
Accept H_0 if $\bar{X} > 0.45$; calculate α

- $\bar{X} > 0.45 \Leftrightarrow \sum_{i=1}^{5} X_i > 5 \cdot 0.45 = 2.25$
  Let $Y = \sum_{i=1}^{5} X_i$ (sum of independent exponentially distributed RV's)

$\Rightarrow$ $Y \sim \mathrm{Erl}(\lambda = 0, n = 5)$ - under $H_0 : E(Y) = 5/2$

- Apparently:

Example 2 (a) – from last lecture

$X_1, \ldots, X_5$ IID, $\sim \mathrm{Exp}(\lambda)$, λ unknown
$H_0 : \lambda = 2$ vs. $H_1 : \lambda > 2$
(So under $H_0 : \mu = E(X_i) = \frac{1}{2}$)
Accept H_0 if $\bar{X} > 0.45$; calculate α

- $\bar{X} > 0.45 \Leftrightarrow \sum_{i=1}^{5} X_i > 5 \cdot 0.45 = 2.25$
  Let $Y = \sum_{i=1}^{5} X_i$ (sum of independent exponentially distributed RV's)

$\Rightarrow$ $Y \sim \mathrm{Erl}(\lambda = 0, n = 5)$ - under $H_0 : E(Y) = 5/2$

- Apparently: $\alpha = P(Y \leq 2.25)$

Example 2 (a) – from last lecture

$X_1, \ldots, X_5$ IID, $\sim \mathrm{Exp}(\lambda)$, λ unknown
$H_0 : \lambda = 2$ vs. $H_1 : \lambda > 2$
(So under $H_0 : \mu = E(X_i) = \frac{1}{2}$)
Accept H_0 if $\bar{X} > 0.45$; calculate α

- $\bar{X} > 0.45 \Leftrightarrow \sum_{i=1}^{5} X_i > 5 \cdot 0.45 = 2.25$
  Let $Y = \sum_{i=1}^{5} X_i$ (sum of independent exponentially distributed RV's)

$\Rightarrow$ $Y \sim \mathrm{Erl}(\lambda = 0, n = 5)$ - under $H_0 : E(Y) = 5/2$

- Apparently: $\alpha = P(Y \leq 2.25) = \int_0^{2.25} \frac{\lambda^n x^{n-1}}{(n-1)!} \exp(-\lambda x) \, \mathrm{d}x$

Example 2 (a) – from last lecture

$X_1, \dots, X_5$ IID, $\sim \mathrm{Exp}(\lambda)$, λ unknown
$H_0 : \lambda = 2$ vs. $H_1 : \lambda > 2$
(So under $H_0 : \mu = E(X_i) = \frac{1}{2}$)
Accept H_0 if $\bar{X} > 0.45$; calculate α

- $\bar{X} > 0.45 \Leftrightarrow \sum_{i=1}^{5} X_i > 5 \cdot 0.45 = 2.25$
  Let $Y = \sum_{i=1}^{5} X_i$ (sum of independent exponentially distributed RV's)

$\Rightarrow$ $Y \sim \mathrm{Erl}(\lambda = 0, n = 5)$ - under $H_0 : E(Y) = 5/2$

- Apparently: $\alpha = P(Y \leq 2.25) = \int_0^{2.25} \frac{\lambda^n x^{n-1}}{(n-1)!} \exp(-\lambda x)\,\mathrm{d}x =$
  $\int_0^{2.25} \frac{2^5 x^4}{4!} \exp(-2x)\,\mathrm{d}x = \frac{4}{3} \int_0^{2.25} x^4 \exp(-2x)\,\mathrm{d}x$

Example 2 (a) – from last lecture

$X_1, \ldots, X_5$ IID, $\sim \mathrm{Exp}(\lambda)$, λ unknown
$H_0 : \lambda = 2$ vs. $H_1 : \lambda > 2$
(So under $H_0 : \mu = E(X_i) = \frac{1}{2}$)
Accept H_0 if $\bar{X} > 0.45$; calculate α

- $\bar{X} > 0.45 \Leftrightarrow \sum_{i=1}^{5} X_i > 5 \cdot 0.45 = 2.25$
  Let $Y = \sum_{i=1}^{5} X_i$ (sum of independent exponentially distributed RV's)

$\Rightarrow$ $Y \sim \mathrm{Erl}(\lambda = 0, n = 5)$ - under H_0 : $E(Y) = 5/2$

- Apparently: $\alpha = P(Y \leq 2.25) = \int_0^{2.25} \frac{\lambda^n x^{n-1}}{(n-1)!} \exp(-\lambda x)\,\mathrm{d}x =$
  $\int_0^{2.25} \frac{2^5 x^4}{4!} \exp(-2x)\,\mathrm{d}x = \frac{4}{3} \int_0^{2.25} x^4 \exp(-2x)\,\mathrm{d}x = \ldots \approx 0.4679$

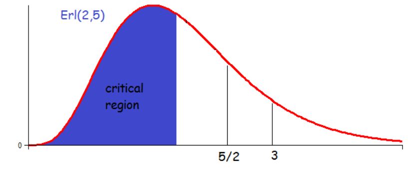
Example 2 (b) – from last lecture

Example 2 (b) – from last lecture

We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?

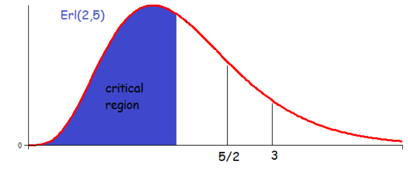
Example 2 (b) – from last lecture

We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?



Example 2 (b) – from last lecture

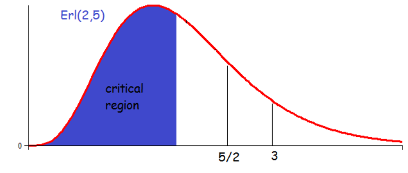
We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?



$E(Y) = 2\frac{1}{2}$

Example 2 (b) – from last lecture

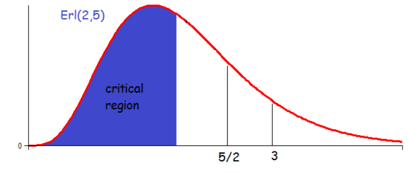
We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?



$E(Y) = 2\frac{1}{2}$
Realization: $y = 3 \Rightarrow$ Corresponds to a λ-value of $\frac{5}{3} < 2$,

Example 2 (b) – from last lecture

We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?

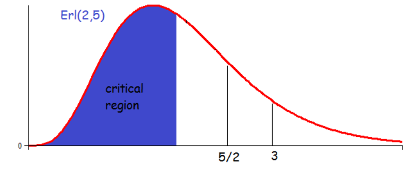


$E(Y) = 2\frac{1}{2}$

Realization: $y = 3 \Rightarrow$ Corresponds to a λ-value of $\frac{5}{3} < 2$, but $H_1 : \lambda > 2$

Example 2 (b) – from last lecture

We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?

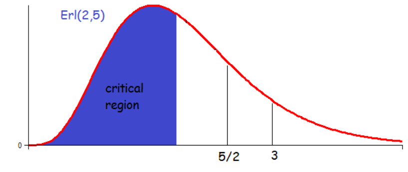


$E(Y) = 2\frac{1}{2}$

Realization: $y = 3 \Rightarrow$ Corresponds to a λ-value of $\frac{5}{3} < 2$, but $H_1 : \lambda > 2 \Rightarrow$ Never reject H_0

Example 2 (b) – from last lecture

We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?

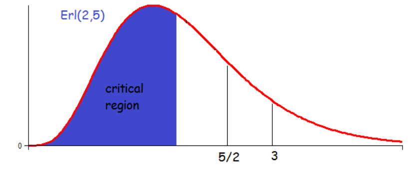


$E(Y) = 2\frac{1}{2}$
Realization: $y = 3 \Rightarrow$ Corresponds to a λ-value of $\frac{5}{3} < 2$, but $H_1 : \lambda > 2 \Rightarrow$
Never reject H_0

Example 2 (c) – from last lecture

Example 2 (b) – from last lecture

We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?



$E(Y) = 2\frac{1}{2}$

Realization: $y = 3 \Rightarrow$ Corresponds to a λ-value of $\frac{5}{3} < 2$, but $H_1 : \lambda > 2 \Rightarrow$

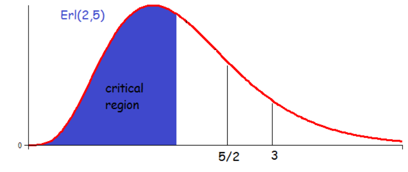
Never reject H_0

Example 2 (c) – from last lecture

Suppose $y = \sum_{i=1}^{5} x_i = 2$

Example 2 (b) – from last lecture

We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?



$E(Y) = 2\frac{1}{2}$

Realization: $y = 3 \Rightarrow$ Corresponds to a λ-value of $\frac{5}{3} < 2$, but $H_1 : \lambda > 2 \Rightarrow$

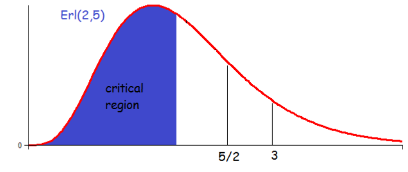
Never reject H_0

Example 2 (c) – from last lecture

Suppose $y = \sum_{i=1}^{5} x_i = 2 < 2.5$

Example 2 (b) – from last lecture

We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?



$E(Y) = 2\frac{1}{2}$

Realization: $y = 3 \Rightarrow$ Corresponds to a λ-value of $\frac{5}{3} < 2$, but $H_1 : \lambda > 2 \Rightarrow$

Never reject H_0

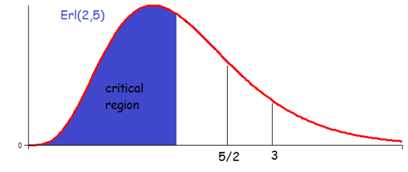
Example 2 (c) – from last lecture

Suppose $y = \sum_{i=1}^{5} x_i = 2 < 2.5$

For what α do we accept H_0 (i.e. what is the P-value for $y = 2$?)

Example 2 (b) – from last lecture

We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?



$E(Y) = 2\frac{1}{2}$

Realization: $y = 3 \Rightarrow$ Corresponds to a λ-value of $\frac{5}{3} < 2$, but $H_1 : \lambda > 2 \Rightarrow$

Never reject H_0

Example 2 (c) – from last lecture

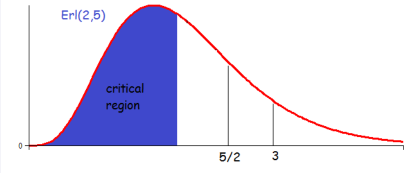
Suppose $y = \sum_{i=1}^{5} x_i = 2 < 2.5$

For what α do we accept H_0 (i.e. what is the P-value for $y = 2$?)

- P-value: $P(Y \leq 2 | H_0$ is true)

Example 2 (b) – from last lecture

We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?



$E(Y) = 2\frac{1}{2}$

Realization: $y = 3 \Rightarrow$ Corresponds to a λ-value of $\frac{5}{3} < 2$, but $H_1 : \lambda > 2 \Rightarrow$

Never reject H_0

Example 2 (c) – from last lecture

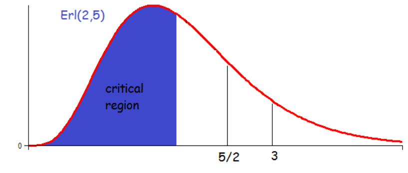
Suppose $y = \sum_{i=1}^{5} x_i = 2 < 2.5$

For what α do we accept H_0 (i.e. what is the P-value for $y = 2$?)

- P-value: $P(Y \leq 2 | H_0 \text{ is true}) = P\left(Y \leq 2 | Y \sim \mathrm{Erl}(\lambda = 2, n = 5)\right)$

Example 2 (b) – from last lecture

We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?



$E(Y) = 2\frac{1}{2}$
Realization: $y = 3 \Rightarrow$ Corresponds to a λ-value of $\frac{5}{3} < 2$, but $H_1 : \lambda > 2 \Rightarrow$
Never reject H_0

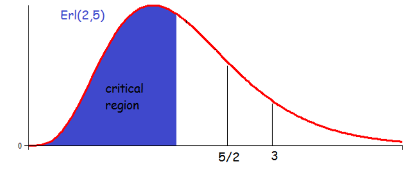
Example 2 (c) – from last lecture

Suppose $y = \sum_{i=1}^{5} x_i = 2 < 2.5$
For what α do we accept H_0 (i.e. what is the P-value for $y = 2$?)

- P-value: $P(Y \leq 2 | H_0 \text{ is true}) = P\left(Y \leq 2 | Y \sim \mathrm{Erl}(\lambda = 2, n = 5)\right)$
  $= \frac{4}{3} \int_0^2 x^4 \exp(-2x)\,\mathrm{d}x$

Example 2 (b) – from last lecture

We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?



$E(Y) = 2\frac{1}{2}$

Realization: $y = 3 \Rightarrow$ Corresponds to a λ-value of $\frac{5}{3} < 2$, but $H_1 : \lambda > 2 \Rightarrow$ Never reject H_0

Example 2 (c) – from last lecture

Suppose $y = \sum_{i=1}^{5} x_i = 2 < 2.5$

For what α do we accept H_0 (i.e. what is the P-value for $y = 2$?)

- P-value: $P(Y \leq 2 | H_0 \text{ is true}) = P\left(Y \leq 2 | Y \sim \mathrm{Erl}(\lambda = 2, n = 5)\right)$
  $= \frac{4}{3} \int_0^2 x^4 \exp(-2\,x)\,\mathrm{d}x = \ldots = 1 - 34 \cdot \frac{1}{3} \cdot \exp(-4) = 0.3712$
  (probably accept H_0)

Various exercises, such as:

Various exercises, such as:

- $X_1,\ldots,X_5$ IID $\mathcal{N}(\mu,\sigma)$, realization: $\{x_1,\ldots,x_5\} = \{3.5, 5.7, 1.2, 6.8, 7.1\}$ $\Rightarrow \bar{x} = 4.86$, $s = 2.487$
  - **(a)** If μ is unknown and $\sigma = 1$, test $H_0 : \mu = 4$ vs. $H_1 : \mu > 4$. Use a P-value.
  - **(b)** If μ is unknown and σ is unknown, test $H_0 : \mu = 4$ vs. $H_1 : \mu > 4$. Use a significance level of 0.10.

Various exercises, such as:

- $X_1, \ldots, X_5$ IID $\mathcal{N}(\mu, \sigma)$, realization: $\{x_1, \ldots, x_5\} = \{3.5, 5.7, 1.2, 6.8, 7.1\}$ $\Rightarrow \bar{x} = 4.86$, $s = 2.487$
  - **(a)** If μ is unknown and $\sigma = 1$, test $H_0 : \mu = 4$ vs. $H_1 : \mu > 4$. Use a P-value.
  - **(b)** If μ is unknown and σ is unknown, test $H_0 : \mu = 4$ vs. $H_1 : \mu > 4$. Use a significance level of 0.10.
- pages 334–335: Exercises 10.6, 10.7

Various exercises, such as:

- $X_1, \ldots, X_5$ IID $\mathcal{N}(\mu, \sigma)$, realization: $\{x_1, \ldots, x_5\} = \{3.5, 5.7, 1.2, 6.8, 7.1\}$ $\Rightarrow \bar{x} = 4.86$, $s = 2.487$
  - **(a)** If μ is unknown and $\sigma = 1$, test $H_0 : \mu = 4$ vs. $H_1 : \mu > 4$. Use a P-value.
  - **(b)** If μ is unknown and σ is unknown, test $H_0 : \mu = 4$ vs. $H_1 : \mu > 4$. Use a significance level of 0.10.
- pages 334–335: Exercises 10.6, 10.7
- Try all odd exercises on pages 334–335