Lecture 18: Last (but interesting) examples in hypothesis testing, Goodness-of-fit testing, Preparation for Exam

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Statistics (MAT1003)

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Outline

book: Chapter 10

And now . . .

1. Last examples in hypothesis testing
2. Goodness-of-fit testing
3. Preparation for Exam 1

Extension of last example from Wednesday

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p- fraction of adults that are colleague graduates

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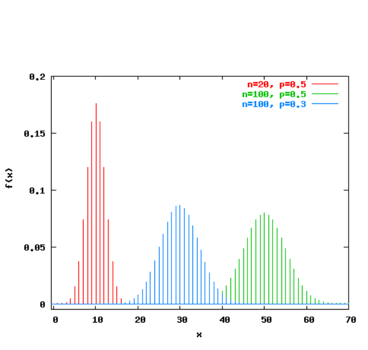

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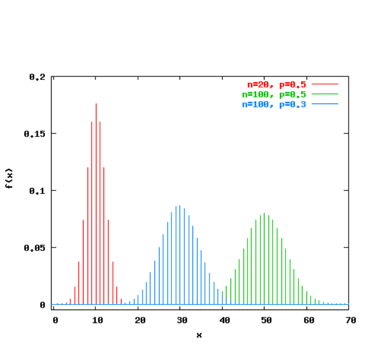

$B(n,p) \approx \mathcal{N}(\mu,\sigma)$ with $\mu = np$, $\sigma = \sqrt{np(1-p)}$,
approximation good if $np > 5$, $n(1-p) > 5$

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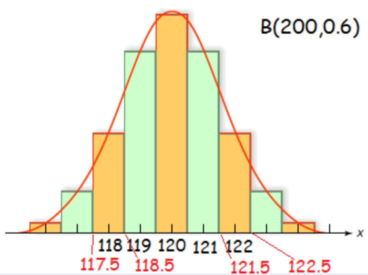

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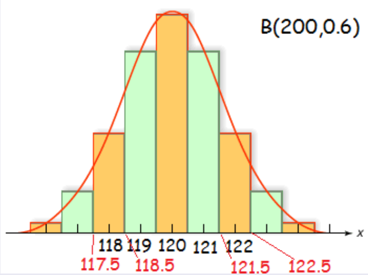

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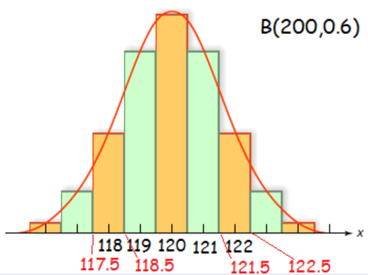

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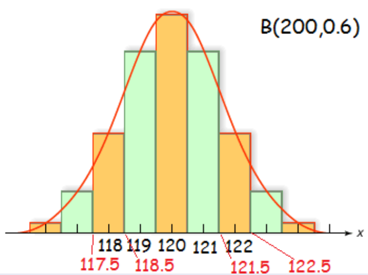

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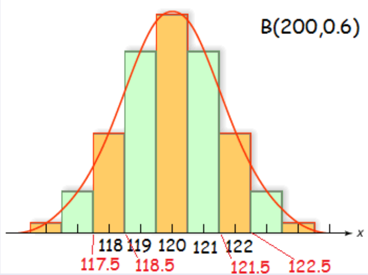

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Exercise 10.7(a) (page 335)

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$H_0 : p = 0.6$, $X \sim \mathcal{B}(200, 0.6) \approx \mathcal{N}(\mu = 200 \cdot 0.6, \sigma = \sqrt{200 \cdot 0.6 \cdot 0.4})$

Exercise 10.7(a) (page 335)

$H_0 : p = 0.6$, $X \sim \mathcal{B}(200, 0.6) \approx \mathcal{N}(\mu = 200 \cdot 0.6, \sigma = \sqrt{200 \cdot 0.6 \cdot 0.4})$
Task: If we accept H_0 if $110 \leq X \leq 130$, compute α

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Solution- with continuity correction $109.5 \leq X \leq 130.5$

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$\beta = P(109.5 \leq X \leq 130.5 | X \sim \mathcal{N}(100, \sqrt{50}))= P(\frac{109.5-100}{\sqrt{50}} \leq Z \leq$
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Under $H_1 : p = 0.7$, we have $X \sim \mathcal{B}(200, 0.7) \approx \mathcal{N}(140, \sqrt{42})$ and
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This can be used for hypotheses concerning σ :

- $H_0 : \sigma^2 = 25$ vs. $\sigma^2 \neq 25$
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Example

Let $X_1, \dots X_5$ IID, $\mathcal{N}(\mu, \sigma)$, μ, σ unknown. Realization:
$\{3.5, 5.7, 1.2, 6.8, 7.1\} \Rightarrow \bar{x} = 4.86, s = 2.487$

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$\{3.5, 5.7, 1.2, 6.8, 7.1\} \Rightarrow \bar{x} = 4.86, s = 2.487$
Test $H_0 : \sigma = 5$ vs. $H_1 : \sigma < 5$, use $\alpha = 0.10$

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- As $\underline{\chi}^2 \stackrel{\text{def}}{=} \frac{(n-1)\, s^2}{\sigma^2} \sim \underline{\chi}^2_{n-1}$, small values of s suggest that H_0 should be rejected
- $P(\underline{\chi}^2 \leq \tilde{\chi}^2_4) = 0.10 \Rightarrow \tilde{\chi}^2_4 = 1.064$ (Table A5)

Hypotheses concerning σ

$X_1, \ldots X_n$ IID $\mathcal{N}(\mu, \sigma)$
$\Rightarrow \underline{\chi}^2 \stackrel{\text{def}}{=} \frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$
This can be used for hypotheses concerning σ :

- $H_0 : \sigma^2 = 25$ vs. $\sigma^2 \neq 25$
- $H_0 : \sigma^2 = 25$ vs. $\sigma^2 < 25$ ($\sigma^2 > 25$)

Example

Let $X_1, \ldots X_5$ IID, $\mathcal{N}(\mu, \sigma)$, μ, σ unknown. Realization:
$\{3.5, 5.7, 1.2, 6.8, 7.1\} \Rightarrow \bar{x} = 4.86, s = 2.487$
Test $H_0 : \sigma = 5$ vs. $H_1 : \sigma < 5$, use $\alpha = 0.10$
Solution:

- As $\underline{\chi}^2 \stackrel{\text{def}}{=} \frac{(n-1)\,s^2}{\sigma^2} \sim \underline{\chi}^2_{n-1}$, small values of s suggest that H_0 should be rejected
- $P(\underline{\chi}^2 \leq \tilde{\chi}^2_4) = 0.10 \Rightarrow \tilde{\chi}^2_4 = 1.064$ (Table A5)
- Under $H_0 : P(4\,s^2/25 \leq 1.064) = 0.10 \Rightarrow P(s \leq 2.579) = 0.1$

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- Under $H_0 : P(4\,s^2/25 \leq 1.064) = 0.10 \Rightarrow P(s \leq 2.579) = 0.1$
- Realization: $s = 2.487 \Rightarrow$ reject H_0

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Test $H_0 : \sigma = 4$ vs. $H_1 : \sigma \neq 4$, use $\alpha = 0.10$

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Test $H_0 : \sigma = 4$ vs. $H_1 : \sigma \neq 4$, use $\alpha = 0.10$

- $\underline{\chi}^2 \stackrel{\text{def}}{=} \frac{4\, s^2}{16} \sim \underline{\chi}^2_4$

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- $\underline{\chi}^2 \stackrel{\text{def}}{=} \frac{4\,s^2}{16} \sim \underline{\chi}^2_4$
- 2-sided test: Find $\tilde{\chi}$ and $\hat{\chi}$ such that $P(\underline{\chi}^2 \leq \tilde{\chi}) = 0.05$ & $P(\underline{\chi}^2 \leq \hat{\chi}) = 0.95 \Rightarrow \tilde{\chi} = 0.711, \hat{\chi} = 9.488$

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- Hence
  $P(0.711 \leq s^2/4 \leq 9.488) = 0.90 \Leftrightarrow P(1.688 \leq s \leq 6.161) = 0.9$

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- Realization: $s = 2.487 \Rightarrow$ accept H_0

And now . . .

1. Last examples in hypothesis testing
2. **Goodness-of-fit testing**
3. Preparation for Exam 1

What is it?

What is it?

A test that checks if a population has a particular probability distribution

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Data: 100 tosses of a coin: 63 H, 37 T. Question: Is this coin fair? Use $\alpha = 0.05$

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Solution :

class	O_i	E_i
H	63	50
T	37	50

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, O_i: observed frequency of class i (in this case H/T), E_i : expected frequency under H_0 : the coin is fair

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- We can now use the following statistics: $\underline{\chi}^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \sim \underline{\chi}^2_{k-1}$, $i =$ class index, $k = \sharp$ classes (2 in this case)

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- Reject H_o if $\underline{\chi}^2$ is too big (one-sided test)
- Make sure that each E_i is at least equal to 5!! (otherwise denominator too small)

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- In the example $P(\underline{\chi}^2 \geq \chi) = 0.05 \Rightarrow \chi = 3.841 \Rightarrow$ critical region:$[3.841, \infty)$

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Example

Data: 100 tosses of a coin: 63 H, 37 T. Question: Is this coin fair? Use $\alpha = 0.05$

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- Reject H_o if $\underline{\chi}^2$ is too big (one-sided test)
- Make sure that each E_i is at least equal to 5!! (otherwise denominator too small)
- In the example $P(\underline{\chi}^2 \geq \chi) = 0.05 \Rightarrow \chi = 3.841 \Rightarrow$ critical region:$[3.841, \infty)$
- Realization: $\underline{\chi}^2 = \frac{(63-50)^2}{50} + \frac{(37-50)^2}{50} = 6.76 \Rightarrow$ reject H_0

And now . . .

- We will go now through typical exam exercises
- Test exam + formula sheet + tables that will be used at the exam will appear on ELEUM today
- Please try it at home within 2 hours and bring your solutions on Tuesday
- Also last homework can help :-)
- Allowed at the exam: Simple calculator