Lecture 2: Concepts of Probability Theory

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Statistics (MAT1003)

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Outline

1. **Concepts from Probability Theory**
   - Experiment, Sample Space, Event
   - Complement of an event, intersection of events, disjoint/mutually exclusive events, union of events
2. **Probabilities**
   - Probability measure, probability of an event
   - Examples
   - Calculating probabilities
3. **Random variables**

And now . . .

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   - Probability measure, probability of an event
   - Examples
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3. **Random variables**

book: Sections 2.1, 2.2

Experiment

Throw a single die

Draw one card out of a deck

Sample Space (assumption: finite & countable)

$S = \{1, 2, 3, 4, 5, 6\}$

$S = \{\spadesuit 2, \spadesuit 3, \ldots, \heartsuit \mathrm{K}, \heartsuit \mathrm{A}\}$

Event

Event A : The outcome is a multiple of 3, i.e. A={3,6}

C : A queen, i.e., $C = \{\clubsuit \mathrm{Q}, \diamondsuit \mathrm{Q}, \heartsuit \mathrm{Q}, \spadesuit \mathrm{Q}\}$

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The set of possible outcomes of an experiment (denoted S)

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Experiment, Sample Space, Event

Experiment

Sample Space (assumption: finite & countable)

Event

D : three times the same number, i.e.
$D = \{111, 222, 333, 444, 555, 666\}$

Experiment

Throwing a coin twice

Sample Space (assumption: finite & countable)

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$D = \{111, 222, 333, 444, 555, 666\}$

Experiment

Throwing a coin twice

Sample Space (assumption: finite & countable)

$S = \{HH, HT, TH, TT\}$ $2 \cdot 2$ options

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D : three times the same number, i.e.
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Experiment

Throwing a coin twice

Sample Space (assumption: finite & countable)

$S = \{HH, HT, TH, TT\}$ $2 \cdot 2$ options

Event

Event B : 2 tails, i.e. B={TT}

D : three times the same number, i.e.
$D = \{111, 222, 333, 444, 555, 666\}$

Experiment

Throwing a coin twice

Throwing a die 3 times

Sample Space (assumption: finite & countable)

$S = \{HH, HT, TH, TT\}$ $2 \cdot 2$ options

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Event B : 2 tails, i.e. B={TT}

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Experiment

Throwing a coin twice

Throwing a die 3 times

Sample Space (assumption: finite & countable)

$S = \{HH, HT, TH, TT\}$ $2 \cdot 2$ options

$S = \{111, 112, 113, \ldots, 666\}$ - $66 \cdot 6 = 216$ options

Event

Event B : 2 tails, i.e. B={TT}

D : three times the same number, i.e.
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Experiment

Throwing a coin twice

Throwing a die 3 times

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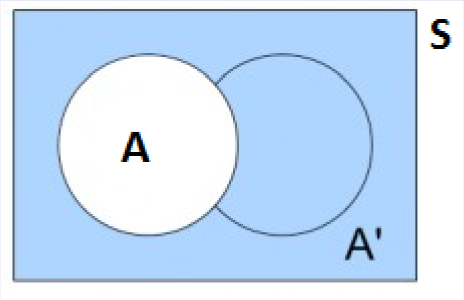
Events are sets, we can therefore talk about:

Complement of an event A: event A’



Example:
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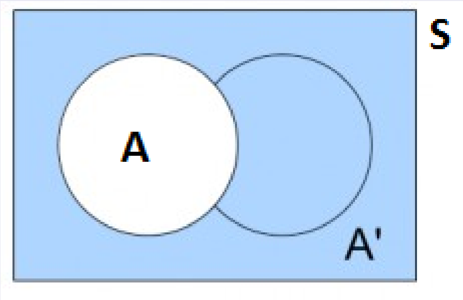

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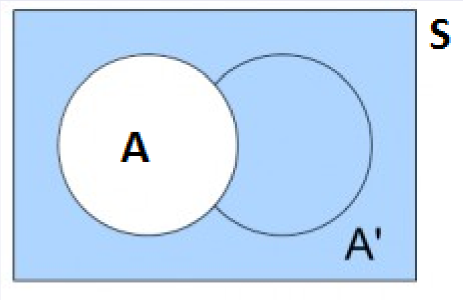

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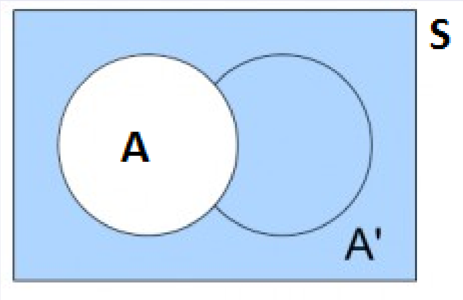

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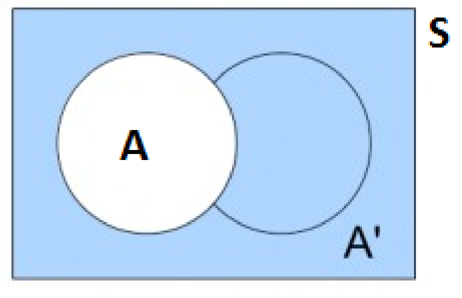

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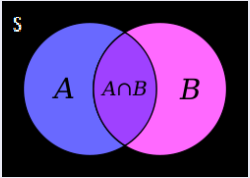

Example: throwing a die

$S = \{1, 2, 3, 4, 5, 6\}$, $A = \{1, 3, 5\}$, $B = \{3, 6\}$, $C = \{4\}$

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$A \cap C = \emptyset \Rightarrow A$ and C and disjoint (book: “mutually exclusive”)

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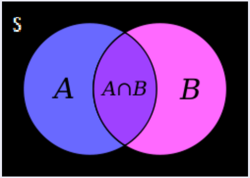

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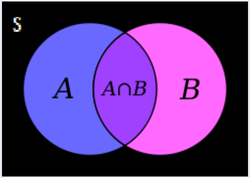

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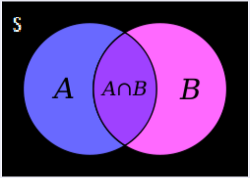

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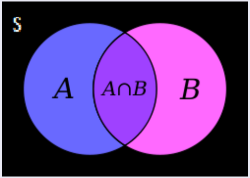

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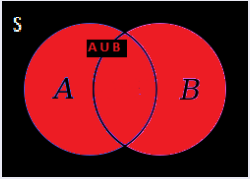

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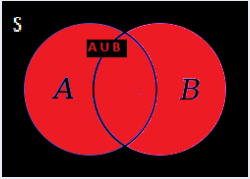

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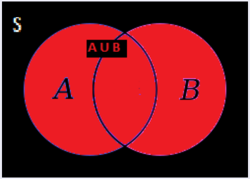

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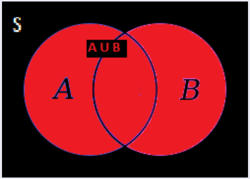

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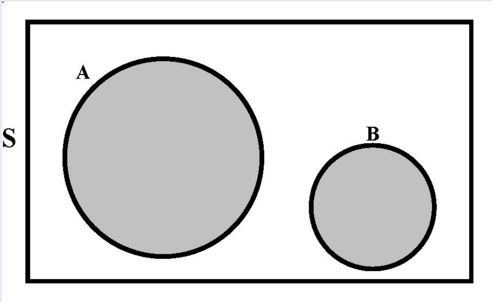
Disjoint sets ("mutually exclusive")

$A \cap B = \emptyset$



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book: Section 2.4

Probability measure

A probability measure is a function that assigns to each element of S a number in [0,1] (the probability), such that the sum of these probabilities is 1

Probability of an event

The probability of event A is the sum of the probabilities of the elements in A (notation: $P(A)$)

$P(S) = 1$, $P(\emptyset) = 0$, $0 \leq P(A) \leq 1$ for each event A

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Examples

Fair die

$S = \{1, 2, 3, 4, 5, 6\}$, $P(\{1\}) = P(\{2\}) = \ldots = P(\{6\}) = \frac{1}{6}$

Unfair die

$S = \{1, 2, 3, 4, 5, 6\}$, $P(\{1\}) = P(\{2\}) = \ldots = P(\{5\}) = \frac{1}{10}$,
$P(\{6\}) = \frac{1}{2}$

5 apples, 4 bananas

$S = \{A_1, A_2, A_3, A_4, A_5, B_1, B_2, B_3, B_4\}$,
$P(\{A_1\}) = \ldots = P(\{B_4\}) = \frac{1}{9}$
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Examples

5 apples, 4 bananas

- Select 2 pieces, what is the probability of selecting one apple and one banana?
- $S = \{A_1, A_2\}, \{A_1, A_3\}, \ldots, \{B_3, B_4\}$
- selection of 1 apple: 5 ways, selection of 1 banana: 4 ways
- selection of 1 apple and 1 banana: $5 \cdot 4 = 20$ ways
- total number of selecting 2 pieces - combination 9 choose 2: $\binom{9}{2} = \frac{9!}{7!\,2!} = 36$
- $P(\text{1 apple and 1 banana}) = \frac{20}{36} = \frac{5}{9} = \boxed{\frac{5}{9}}$
- $P(\text{2 apples}), P(\text{2 bananas}) = ?$

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- $P(\text{1 apple and 1 banana}) = \frac{20}{36} = \frac{5}{9} = \frac{|A|}{|S|}$
- P(2 apples), P(2 bananas) =?

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5 apples, 4 bananas

- Select 2 pieces, what is the probability of selecting one apple and one banana?
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- selection of 1 apple: 5 ways, selection of 1 banana: 4 ways
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- $P(A) = \frac{|A|}{|S|}$ if each element of S has the same probability (Thm. 2.9)
- For any events A and B: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (Thm. 2.10)
- For disjoint events A and B: $P(A \cup B) = P(A) + P(B)$ (Thm. 2.10)
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Example: Throw a die

$S = \{1, 2, 3, 4, 5, 6\}$, events: $A = \{1\}$, $B = \{2, 4, 6\}$, $C = \{1, 2, 3\}$

$P(A) = ?$, $P(B \cup C) = ?$, $P(A \cup B) = ?$, $P(C') = ?$

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And now . . .

Book: Section 3.1

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X is a random variable for the sample space S if it assigns a real number to each element of S, $X : S \to \mathbb{R}$

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Throwing a die until a 6 comes up
$S = \{6, N6, NN6, NNN6, \ldots\}$
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