Lecture 3: Probabilities and distributions (Part 1)

Kateřina Staňková

Statistics (MAT1003)

April 16, 2012

Outline

And now . . .

book: Section 3.1, recommended exercises: 3.1, 3.3

Random variable

X is a random variable for the sample space S if it assigns a real number to each element of S, $X : S \to \mathbb{R}$

Example 1

$S = \{1, 2, 3, 4\}$
X :square: $\forall s \in S : X(s) = s^2$
$Y : 2 \cdot s$ if s is odd, $\frac{s}{2}$ if s is even
$Y(1) = 2$, $Y(2) = 1$, $Y(3) = 6$, $Y(4) = 2$

Example 2

Throwing a die until a 6 comes up
$S = \{6, N6, NN6, NNN6, \ldots\}$
X : number of throws required
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Discrete vs. continuous random variable

X is a discrete if its set of possible outcomes is finite and countable
Otherwise X is continuous

Example

$S = [1, 2]$
$X(s) = \frac{1}{s}\ \forall s \in S \Rightarrow X \in [\frac{1}{2}, 1]$ is continuous

$$Y(s) = \lfloor 2\,s \rfloor\ \forall s \in S \Rightarrow Y(s) = \begin{cases} 2, & \text{if} \quad s \in [1, 1\,\frac{1}{2}) \\ 3, & \text{if} \quad s \in [1\,\frac{1}{2}, 2) \\ 4, & \text{if} \quad s = 2 \end{cases}$$

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Example: Throwing a die

$S = \{1, 2, 3, 4, 5, 6\}$, $X(s) : \lfloor \frac{s}{2} \rfloor$, where $s = \sharp$ on the die

$f(0) = P(X = 0) = P(\{1\}) = \frac{1}{6}$

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$f(x) = P(X = x)$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$
$F(x) = P(X \leq x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{5}{6}$	1

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Cumulative distribution $F(x)$

$$P(X \leq x) = F(x) = \sum_{y \leq x} f(y)$$

Exercise - from Example 3.6

A shipment of 20 similar laptop computers to a retail outlet contains 3 that are defective. If a school makes a random purchase of 2 of these computers

(a) find the probability distribution for the number of deffectives.

(b) find the cumulative distribution function $F(x)$

Exercise

Let X be a random variable giving the number of heads minus the number of tails in three tosses of a coin. Find

(a) the probability distribution of this random variable

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Example

$S = [0, 1]$, $X(s) = s$, and suppose that all outcomes are "equally likely".
What is the probability $P(X = \frac{1}{2})$?

book: Section 3.3, recommended exercises: 3.9, 3.17, 3.29, 3.31, 3.33

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book: Section 3.3, recommended exercises: 3.9, 3.17, 3.29, 3.31, 3.33

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Cumulative distribution

$F(x) = P(X \leq x)$ - well defined this way

The probability distribution (density function)

For continuous RV defined as $f(x) = F'(x)$

Example

$$F(x) = \begin{cases} 0, & \text{if} \quad x \leq 0, \\ x, & \text{if} \quad 0 \leq x \leq 1, \\ 1, & \text{if} \quad x \geq 1 \end{cases}$$

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Exercise 3.9

And now . . .

Bring to the lecture on April 23

1. We have a density function of an random variable X, defined as

$$f(x) = \begin{cases} \frac{7-2x}{2}, & 2 < x < 3, \\ 0, & \text{else} \end{cases}$$

   Calculate:
   (a) $P(x \leq 2\frac{1}{2})$
   (b) $F(x)$
2. Book (pp. 91-92): Exercises 3.6, 3.24
3. Book (pp. 70-71): Exercises 2.80, 2.84
4. Book (pp. 104-105): Exercises 3.40, 3.44