Lecture 5: Probabilities and distributions (Part 3)

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Statistics (MAT1003)

April 19, 2012

Outline

And now . . .

News

- Homework deadline: almost one day later: April 24, 13.35; also new homework will be assigned on Tuesday
- Absence: Measured from $\frac{2}{3}$ of classes
- April 23: Exercise session $\Rightarrow$ lot of theory today

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book: Section 3.4

Continuous Joint Probability Distribution

Let X be a continuous RV with probability density function f. Then

$$f(x) = F'(x) = \frac{\mathrm{d}}{\mathrm{d}x} P(X \leq x) = \lim_{\Delta x \to 0} \frac{P(X \leq x + \Delta x) - P(X \leq x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{P(x \leq X \leq x + \Delta x)}{\Delta x}$$

Analogously for 2 continuous RV's X and Y :

$$f(x, y) = \lim_{(\delta,\epsilon)\to(0,0)} \frac{P(x \leq X \leq x + \delta, y \leq Y \leq y + \epsilon)}{\delta\epsilon}$$

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Properties of Continuous Joint PD

1. $f(x, y) \geq 0$ for all $x, y \in \mathbb{R}$
2. $\iint_{\mathbb{R}^2} f(x,y)dxdy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dxdy = 1$
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Example 1

X, Y RV's with

$$f(x, y) = \begin{cases} \frac{y}{2x^2}, & \frac{1}{2} < x < 1, 0 < y < 2 \\ 0, & \text{elsewhere} \end{cases}$$

(a) Validate condition 2. (b) Calculate $P(X \geq \frac{3}{5})$

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Example 1(a): Verify $\int\int_{\mathbb{R}^2} f(x,y)\mathrm{d}x\mathrm{d}y = 1$

($f(x,y) = \frac{y}{2x^2}$, if $\frac{1}{2} < x < 1$, $0 < y < 2$ and 0 elsewhere)

$$\int\int_{\mathbb{R}^2} f(x,y)\mathrm{d}x\mathrm{d}y = \int_{\frac{1}{2}}^{1}\int_{0}^{2} \frac{y}{2x^2}\mathrm{d}y\mathrm{d}x$$

$$= \int_{\frac{1}{2}}^{1}\left[\frac{y^2}{4x^2}\right]_0^2 \mathrm{d}x = \int_{\frac{1}{2}}^{1}\frac{1}{x^2}\mathrm{d}\,x = -\left[\frac{1}{x}\right]_{\frac{1}{2}}^{1} = 1$$

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Example 1(b): Calculate $P(X \geq \frac{3}{5})$

$$P(X \geq \frac{3}{5}) = \int_{\frac{3}{5}}^{1}\int_{0}^{2}\frac{y}{2x^2}\mathrm{d}y\mathrm{d}x = \int_{\frac{3}{5}}^{1}\frac{1}{x^2}\mathrm{d}\,x = \frac{2}{3}$$

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3. $\int\int_{A} f(x, y)\mathrm{d}x\mathrm{d}y = P\left((x, y) \in A\right)$ for all $A \subseteq \mathbb{R}^2$

Example 2

$X,\ Y$ RV's with

$$f(x, y) = \begin{cases} 10\,x\,y^2, & 0 < x < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

(a) Validate condition 2. (b) Calculate $P(\frac{1}{4} \leq Y \leq \frac{1}{2})$
(c) Calculate $P(\frac{1}{8} \leq X \leq \frac{3}{8}, \frac{1}{4} \leq Y \leq \frac{1}{2})$

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Example 2(a): Verify $\int\int_{\mathbb{R}^2} f(x,y)\,\mathrm{d}x\mathrm{d}y = 1$

$$\left(f(x,y) = \begin{cases} 10\,x\,y^2, & 0 < x < y < 1 \\ 0, & \textbf{elsewhere} \end{cases} \right)$$

Find the proper area:

- Either we notice that x is between 0 and 1 and that y is between x and 1
- Or: y between 0 and 1 and x between 0 and y

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$$\int\int_{\mathbb{R}^2} f(x,y)\mathrm{d}x\mathrm{d}y = \int_0^1 \int_x^1 10\,xy^2 \mathrm{d}y\mathrm{d}x$$

$$= \int_0^1 \int_0^y 10\,x\,y^2\,\mathrm{d}x\mathrm{d}y = 1$$

Example 2(b): Calculate $P(\frac{1}{4} \leq Y \leq \frac{1}{2})$

$$\left(f(x,y) = \begin{cases} 10\,x\,y^2, & 0 < x < y < 1 \\ 0, & \textbf{elsewhere} \end{cases} \right)$$

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$$= \int_{\frac{1}{4}}^{\frac{1}{2}} 5\,y^4\,\mathrm{d}y = \left[y^5\right]_{\frac{1}{4}}^{\frac{1}{2}} = \frac{31}{1024}$$

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Example 2(c): Calculate $P(\frac{1}{8} \leq X \leq 38, \frac{1}{4} \leq Y \leq \frac{1}{2})$

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$$= \int_{\frac{1}{8}}^{\frac{1}{4}} \frac{35}{64}x\,\mathrm{d}x + \int_{\frac{1}{4}}^{\frac{3}{8}} \left(\frac{5}{12}x - \frac{10}{3}x^4\right)\mathrm{d}x$$

$$= \frac{35}{128}\left[x^2\right]_{x=\frac{1}{8}}^{\frac{1}{4}} + \left[\left(\frac{5}{24}x^2 - \frac{2}{3}x^5\right)\right]_{x=\frac{1}{4}}^{\frac{3}{8}} = \frac{1219}{49152}$$

And now . . .

Probability density $g(x)$

Let X and Y be RV's with joint PDF $f(x, y)$. Then

$$g(\tilde{x}) = G'(\tilde{x}) = \lim_{\delta \downarrow 0} \frac{P(\tilde{x} \leq X \leq \tilde{x} + \delta)}{\delta}$$

$P(\tilde{x} \leq X \leq \tilde{x} + \delta)$

$$P(\tilde{x} \leq X \leq \tilde{x} + \delta) = P(\tilde{x} \leq X \leq \tilde{x} + \delta, Y \in \mathbb{R})$$

$$= \int_{-\infty}^{\infty} \underbrace{\int_{x=\tilde{x}}^{\tilde{x}+\delta} f(x, y) \mathrm{d}x}_{\approx \delta f(\tilde{x}, y) \text{ if } \delta \downarrow 0} \mathrm{d}y \approx \int_{-\infty}^{\infty} \delta f(\tilde{x}, y) \, \mathrm{d}y = \delta \int_{-\infty}^{\infty} f(\tilde{x}, y) \, \mathrm{d}y$$

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$P(\tilde{x} \leq X \leq \tilde{x} + \delta)$

$$P(\tilde{x} \leq X \leq \tilde{x} + \delta) = P(\tilde{x} \leq X \leq \tilde{x} + \delta, Y \in \mathbb{R})$$

$$= \int_{-\infty}^{\infty} \underbrace{\int_{x=\tilde{x}}^{\tilde{x}+\delta} f(x, y) \mathrm{d}x}_{\approx \delta f(\tilde{x}, y) \text{ if } \delta \downarrow 0} \mathrm{d}y \approx \int_{-\infty}^{\infty} \delta\, f(\tilde{x}, y)\, \mathrm{d}y = \delta \int_{-\infty}^{\infty} f(\tilde{x}, y)\, \mathrm{d}y$$

Marginal probability density function of X - $g(x)$

$$g(x) = \lim_{\delta \downarrow 0} \frac{P(x \leq X \leq x + \delta)}{\delta} = \int_{-\infty}^{\infty} f(x, y)\, \mathrm{d}y$$

We integrate over the other variable.

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$$f(x,y) = \begin{cases} \frac{y}{2x^2}, & \frac{1}{2} < x < 1, 0 < y < 2 \\ 0, & \text{otherwise} \end{cases}$$

Find marginal density functions $g(x)$, $h(y)$

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And now . . .

book: Section 2.6

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$P(B|A) = \frac{P(\{2\})}{P(\{1,2\})} = \frac{\frac{2}{10}}{\frac{3}{10}} = \frac{2}{3}$ (conditional probability)

Conditional probability

Dependence of events

In the example we have $P(B) = \frac{1}{2}$, $P(B|A) = \frac{2}{3}$

Conditional probability

Probability of event B given A is

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for any two events A and B, as long as $P(A) > 0$.

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A and B are dependent

Independence of *RV*

Since $P(A \cap B) = P(A) \cdot P(B|A)$ and $P(A \cap B) = P(B) \cdot P(A|B)$),
A and B are independent $\Leftrightarrow P(A \cap B) = P(A) \cdot P(B)$

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A and B are called independent if (a) $P(A|B) = P(A)$ or (b) $P(B|A) = P(B)$

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And now . . .

book: Section 3.4

Conditional Probability Distribution

Let X and Y be discrete RV's with joint probability distribution f. The conditional probability of X given that Y has the value y is
$f(x|y) = P(X = x|Y = y) = \frac{P(X=x, Y=y)}{P(Y=y)} = \frac{f(x,y)}{h(y)}$

Example

Throwing 2 dice, X : maximum, Y : minimum. What is conditional probability distribution of X given $Y = 2$?

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(CAH) $P(Y = 2) = \frac{1}{4}$, $x = 3, 4, 5, 6 : f(x, y) = \frac{1}{18}$,
$x = 2 : f(x, y) = \frac{1}{36}$

book: Section 3.4

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x	2	3, 4, 5
$P(X = x\|Y = 2)$	$\frac{\frac{1}{36}}{\frac{1}{4}} = \frac{1}{9}$	$\frac{\frac{1}{18}}{\frac{1}{4}} = \frac{2}{9}$

Continuous = Density

book: Section 3.4

Conditional Probability Distribution

Let X and Y be continuous RV's with joint probability density f. The conditional probability of X given that Y has the value y is

$$
\begin{aligned}
f(x|y) &= \lim_{(\delta,\epsilon)\downarrow 0} \frac{P(x \le X \le x+\delta | y \le Y \le y+\epsilon)}{\delta} \\
&= \lim_{(\delta,\epsilon)\downarrow 0} \frac{P(x \le X \le x+\delta, y \le Y \le y+\epsilon)}{\delta \cdot P(y \le Y \le y+\epsilon)} \\
&= \lim_{(\delta,\epsilon)\downarrow 0} \frac{P(x \le X \le x+\delta, y \le Y \le y+\epsilon)}{\delta\epsilon} \cdot \frac{\epsilon}{P(y \le Y \le y+\epsilon)} \\
&= f(x,y) \cdot \frac{1}{h(y)} = \frac{f(x,y)}{h(y)}, \quad \text{provided } h(y) > 0
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Same formula as for discrete case!

book: Section 3.4

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$$f(x,y) = \begin{cases} 10\,x\,y^2, & ,0 < x < y \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

Calculate conditional density of X, given $Y = \frac{1}{2}$ and calculate $P(X > \frac{1}{3} | Y = \frac{1}{2})$

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$P(X > \frac{1}{3}|Y = \frac{1}{2}) = \int_{\frac{1}{3}}^{\frac{1}{2}} 8\,x\mathrm{d}x = \left[4\,x^2\right]_{x=\frac{1}{3}}^{\frac{1}{2}} = \frac{5}{9}$