Lecture 5: Probabilities and distributions (Part 3)

Kateřina Staňková

Statistics (MAT1003)

April 19, 2012

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Outline



- 2 Continuous Joint PD
- 3 Continuous Marginal Distributions
- Conditional Probabilities
- 5 Conditional Probability Distributions
 - Discrete
 - Continuous = Density

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And now ...



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 Homework deadline: almost one day later: April 24, 13.35; also new homework will be assigned on Tuesday

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- April 23: Exercise session \Rightarrow lot of theory today

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 - Discrete
 - Continuous = Density

book: Section 3.4

Continuous Joint Probability Distribution

Let X be a continuous RV with probability density function f. Then

$$f(x) = F'(x) = \frac{d}{dx} P(X \le x)$$
$$= \lim_{\Delta x \to 0} \frac{P(X \le x + \Delta x) - P(X \le x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{P(x \le X \le x + \Delta x)}{\Delta x}$$

Analogously for 2 continuous RV's X and Y :

$$f(x,y) = \lim_{(\delta,\epsilon)\to(0,0)} \frac{P(x \le X \le x + \delta, y \le Y \le y + \epsilon)}{\delta\epsilon}$$

is the joint probability density function.

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Properties of Continuous Joint PD

- $0 \quad \int \int f(x,y) dx dy = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) dx dy = 1$
- $\bigcirc \int \int f(x, y) dx dy = P((x, y) \in A)$ for all $A \subseteq \mathbb{R}^{2d}$

For 2 continuous RV's X and Y :

$$f(x,y) = \lim_{(\delta,\epsilon)\to(0,0)} \frac{P(x \le X \le x + \delta, y \le Y \le y + \epsilon)}{\delta\epsilon}$$

is the joint probability density function.

Properties of Continuous Joint PD

- $f(x, y) \ge 0$ for all $x, y \in \mathbb{R}$
- 2 $\int_{\mathbb{T}^2} \int f(x,y) dx dy = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x,y) dx dy = 1$

For 2 continuous RV's X and Y :

$$f(x,y) = \lim_{(\delta,\epsilon)\to(0,0)} \frac{P(x \le X \le x + \delta, y \le Y \le y + \epsilon)}{\delta\epsilon}$$

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Properties of Continuous Joint PD

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$$f(x,y) \ge 0$$
 for all $x, y \in \mathbb{R}$

2) $\int_{\mathbb{T}^2} \int f(x, y) dx dy = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y) dx dy = 1$

3)
$$\int \int f(x,y) \mathrm{d}x \mathrm{d}y = P\left((x,y) \in A\right)$$
 for all $A \subseteq \mathbb{R}^2$

For 2 continuous RV's X and Y:

$$f(x,y) = \lim_{(\delta,\epsilon)\to(0,0)} \frac{P(x \le X \le x + \delta, y \le Y \le y + \epsilon)}{\delta\epsilon}$$

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Properties of Continuous Joint PD

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3 $\int_{-\infty}^{\infty} f(x, y) dx dy = P((x, y) \in A)$ for all $A \subseteq \mathbb{R}^2$

Properties of Continuous Joint PD

•
$$f(x, y) \ge 0$$
 for all $x, y \in \mathbb{R}$

$$\int_{\mathbb{R}^2} \int f(x, y) \mathrm{d}x \mathrm{d}y = 1$$

$$\int_{A} \int f(x, y) \mathrm{d}x \mathrm{d}y = P\left((x, y) \in A\right) \text{ for all } A \subseteq \mathbb{R}^{2}$$

Example 1

X, Y RV's with

$$f(x, y) = \begin{cases} \frac{y}{2x^2}, & \frac{1}{2} < x < 1, \ 0 < y < 2\\ 0, & \text{elsewhere} \end{cases}$$

(a) Validate condition 2. (b) Calculate $P(X \ge \frac{3}{2})$

Properties of Continuous Joint PD

•
$$f(x,y) \ge 0$$
 for all $x, y \in \mathbb{R}$

$$\int_{\mathbb{R}^2} \int f(x, y) \mathrm{d}x \mathrm{d}y = 1$$

3
$$\int_{A} \int f(x, y) \mathrm{d}x \mathrm{d}y = P((x, y) \in A)$$
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Example 1

X, Y RV's with

$$f(x, y) = \begin{cases} \frac{y}{2x^2}, & \frac{1}{2} < x < 1, \ 0 < y < 2\\ 0, & \text{elsewhere} \end{cases}$$

(a) Validate condition 2. (b) Calculate $P(X \ge \frac{3}{5})$

Example 1(a): Verify
$$\iint_{\mathbb{R}^2} f(x, y) dx dy = 1$$

 $(f(x, y) = \frac{y}{2x^2}, \text{ if } \frac{1}{2} < x < 1, 0 < y < 2 \text{ and } 0 \text{ elsewhere })$
 $\iint_{\mathbb{R}^2} f(x, y) dx dy = \int_{\frac{1}{2}}^1 \int_0^2 \frac{y}{2x^2} dy dx$
 $= \int_{\frac{1}{2}}^1 \left[\frac{y^2}{4x^2} \right]_0^2 dx = \int_{\frac{1}{2}}^1 \frac{1}{x^2} dx = -\left[\frac{1}{x} \right]_{\frac{1}{2}}^1 = 1$

Example 1(a): Verify $\int_{\mathbb{R}^2} f(x, y) dx dy = 1$ ($f(x, y) = \frac{y}{2x^2}$, if $\frac{1}{2} < x < 1$, 0 < y < 2 and 0 elsewhere)

$$\int_{\mathbb{R}^2} \int f(x, y) dx dy = \int_{\frac{1}{2}}^1 \int_0^2 \frac{y}{2x^2} dy dx$$
$$= \int_{\frac{1}{2}}^1 \left[\frac{y^2}{4x^2}\right]_0^2 dx = \int_{\frac{1}{2}}^1 \frac{1}{x^2} dx = -\left[\frac{1}{x}\right]_{\frac{1}{2}}^1 = 1$$

Example 1(b): Calculate $P(X \ge \frac{3}{5})$

$$P(X \ge \frac{3}{5}) = \int_{\frac{3}{5}}^{1} \int_{0}^{2} \frac{y}{2x^{2}} dy dx = \int_{\frac{3}{5}}^{1} \frac{1}{x^{2}} dx = \frac{2}{3}$$

Properties of Continuous Joint PD

•
$$f(x, y) \ge 0$$
 for all $x, y \in \mathbb{R}$

3)
$$\int_{A} \int f(x, y) dx dy = P((x, y) \in A)$$
 for all $A \subseteq \mathbb{R}^2$

Example 2

X, Y RV's with

$$f(x, y) = \begin{cases} 10 \ x \ y^2, & 0 < x < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

(a) Validate condition 2. (b) Calculate $P(\frac{1}{4} \le Y \le \frac{1}{2})$ (c) Calculate $P(\frac{1}{8} \le X \le \frac{3}{8}, \frac{1}{4} \le Y \le \frac{1}{2})$

Properties of Continuous Joint PD

•
$$f(x, y) \ge 0$$
 for all $x, y \in \mathbb{R}$

$$\int_{\mathbb{D}^2} \int f(x, y) \mathrm{d}x \mathrm{d}y = 1$$

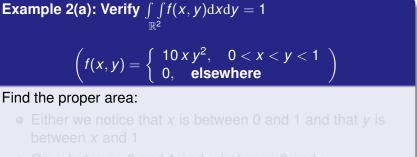
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Example 2

X, Y RV's with

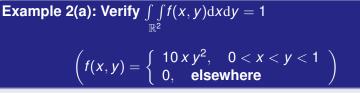
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(a) Validate condition 2. (b) Calculate $P(\frac{1}{4} \le Y \le \frac{1}{2})$ (c) Calculate $P(\frac{1}{8} \le X \le \frac{3}{8}, \frac{1}{4} \le Y \le \frac{1}{2})$



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Or: y between 0 and 1 and x between 0 and y



Find the proper area:

 Either we notice that x is between 0 and 1 and that y is between x and 1

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Example 2(a): Verify
$$\int_{\mathbb{R}^2} \int f(x, y) dx dy = 1$$

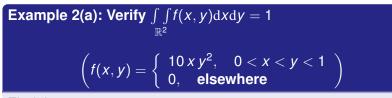
 $\left(f(x, y) = \begin{cases} 10 \ x \ y^2, & 0 < x < y < 1\\ 0, & \text{elsewhere} \end{cases}\right)$

Find the proper area:

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Find the proper area:

- Either we notice that x is between 0 and 1 and that y is between x and 1
- Or: y between 0 and 1 and x between 0 and y

$$\int_{\mathbb{R}^2} \int f(x, y) dx dy = \int_0^1 \int_x^1 10 \, x \, y^2 dy dx$$
$$= \int_0^1 \int_0^y 10 \, x \, y^2 \, dx dy = 1$$

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Example 2(b): Calculate
$$P(\frac{1}{4} \le Y \le \frac{1}{2})$$

 $\begin{pmatrix} f(x,y) = \begin{cases} 10 \times y^2, & 0 < x < y < 1 \\ 0, & \text{elsewhere} \end{cases}$ $\end{pmatrix}$
 $\int_{A} \int f(x,y) dx dy = \int_{\frac{1}{4}}^{\frac{1}{2}} \int_{0}^{y} 10x y^2 dx dy = \int_{\frac{1}{4}}^{\frac{1}{2}} \left[5 x^2 y^2 \right]_{x=0}^{x=y} dy$
 $= \int_{\frac{1}{4}}^{\frac{1}{2}} 5 y^4 dy = \left[y^5 \right]_{\frac{1}{4}}^{\frac{1}{2}} = \frac{31}{1024}$

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Area A: $0 \le x \le y \le 1$ and $\frac{1}{4} \le y \le \frac{1}{2}$
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Example 2(c): Calculate
$$P(\frac{1}{8} \le X \le 38, \frac{1}{4} \le Y \le \frac{1}{2})$$

 $\begin{pmatrix} f(x, y) = \begin{cases} 10 \ x \ y^2, & 0 < x < y < 1 \\ 0, & \text{elsewhere} \end{cases} \end{pmatrix}$

Area A

• $0 \le x \le y, \frac{1}{8} \le x \le \frac{3}{8}, \frac{1}{4} \le y \le \frac{1}{2}$ • If $\frac{1}{8} \le x \le \frac{1}{4}$, then $\frac{1}{4} \le y \le \frac{1}{2}$ and if $\frac{1}{4} \le x \le \frac{3}{8}$, then $x \le y \le \frac{1}{2}$

$$\int_{A} \int f(x, y) \mathrm{d}x \mathrm{d}y = \int_{\frac{1}{8}}^{\frac{1}{4}} \int_{\frac{1}{4}}^{\frac{1}{2}} 10x \, y^2 \mathrm{d}y \mathrm{d}x + \int_{\frac{1}{4}}^{\frac{3}{8}} \int_{x}^{\frac{1}{2}} 10x \, y^2 \mathrm{d}y \mathrm{d}x$$

$$= \int_{\frac{1}{8}}^{\frac{1}{4}} \left[\frac{10}{3} \, x \, y^3 \right]_{y=\frac{1}{4}}^{\frac{1}{2}} \mathrm{d}x + \int_{\frac{1}{4}}^{\frac{3}{8}} \left[\frac{10}{3} \, x \, y^3 \right]_{y=x}^{\frac{1}{2}} \mathrm{d}x$$

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$$\int_{A}^{\frac{1}{4}} \left[10 - x \right]^{\frac{1}{2}} \int_{x}^{\frac{3}{8}} \left[10 - x \right]^{\frac{1}{2}}$$

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$$= \int_{\frac{1}{8}}^{\frac{1}{4}} \frac{35}{64} x \, dx + \int_{\frac{1}{4}}^{\frac{3}{8}} \left(\frac{5}{12} \, x - \frac{10}{3} \, x^4 \right) dx$$
$$= \frac{35}{128} \left[x^2 \right]_{x=\frac{1}{8}}^{\frac{1}{4}} + \left[\left(\frac{5}{24} x^2 - \frac{2}{3} x^5 \right) \right]_{x=\frac{1}{4}}^{\frac{3}{8}} = \frac{1219}{49152}$$

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And now ...



- 2 Continuous Joint PD
- 3 Continuous Marginal Distributions
- Conditional Probabilities
- 5 Conditional Probability Distributions
 - Discrete
 - Continuous = Density

Probability density g(x)

Let X and Y be RV's with joint PDF f(x, y). Then

$$g(ilde{x}) = G'(ilde{x}) = \lim_{\delta \downarrow 0} rac{P(ilde{x} \leq X \leq ilde{x} + \delta)}{\delta}$$

$P(\tilde{x} \le X \le \tilde{x} + \delta)$

$$P(\tilde{x} \le X \le \tilde{x} + \delta) = P(\tilde{x} \le X \le \tilde{x} + \delta, Y \in \mathbb{R})$$

= $\int_{-\infty}^{\infty} \underbrace{\int_{x=\tilde{x}}^{\tilde{x}+\delta} f(x, y) dx}_{\approx \delta f(\tilde{x}, y) dy} dy \approx \int_{-\infty}^{\infty} \delta f(\tilde{x}, y) dy = \delta \int_{-\infty}^{\infty} f(\tilde{x}, y) dy$

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$$g(x) = \lim_{\delta \downarrow 0} \frac{P(x \le X \le x + \delta)}{\delta} = \int_{-\infty}^{\infty} f(x, y) \, \mathrm{d}y$$

We integrate over the other variable.

Marginal probability density function of *Y* - h(x)

$$h(y) = \lim_{\epsilon \downarrow 0} \frac{P(y \le Y \le y + \epsilon)}{\epsilon} = \int_{-\infty}^{\infty} f(x, y) \, \mathrm{d}x$$

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$$f(x, y) = \begin{cases} \frac{y}{2x^2}, & \frac{1}{2} < x < 1, 0 < y < 2\\ 0, & \text{otherwise} \end{cases}$$

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Hence: Integrate over the other variable.

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And now ...

Some News

- 2 Continuous Joint PD
- 3 Continuous Marginal Distributions
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Main Question

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If we already know that an event *A* occurs, what is then the probability that an event *B* occurs?



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$$S = \{1, 2, 3, 4\}, P(\{i\}) = \frac{i}{10}, i = 1, 2, 3, 4$$

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 $B|A$: The event B given A

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But given that A occurs the sample space is actually reduced to

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$$P(B|A) = \frac{P(\{2\})}{P(\{1,2\})} = \frac{\frac{2}{10}}{\frac{3}{10}} = \frac{2}{3} \text{ (conditional probability)}$$

Dependence of events

In the example we have $P(B) = \frac{1}{2}$, $P(B|A) = \frac{2}{3}$

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Probability of event B given A is

$$\mathsf{P}(B|A) = rac{\mathsf{P}(A \cap B)}{\mathsf{P}(A)}$$

for any two events A and B, as long as P(A) > 0.

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for any two events A and B, as long as P(A) > 0.

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And now ...

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- 2 Continuous Joint PD
- 3 Continuous Marginal Distributions
- Conditional Probabilities
- 5 Conditional Probability Distributions
 - Discrete
 - Continuous = Density

book: Section 3.4

Conditional Probability Distribution

Let *X* and *Y* be discrete RV's with joint probability distribution *f*. The conditional probability of *X* given that *Y* has the value *y* is $f(x|y) = P(X = x|Y = y) = \frac{P(X=x, Y=y)}{P(Y=y)} = \frac{f(x,y)}{h(y)}$

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Example

Throwing 2 dice, X : maximum, Y : minimum. What is conditional probability distribution of X given Y = 2?

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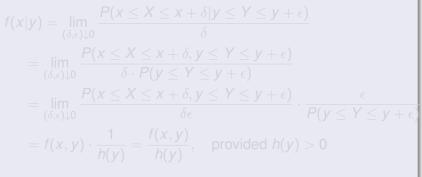
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book: Section 3.4

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Same formula as for discrete case!

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$$f(x|y) = \lim_{(\delta,\epsilon)\downarrow 0} \frac{P(x \le X \le x + \delta | y \le Y \le y + \epsilon)}{\delta}$$

=
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$$\begin{split} f(x|y) &= \lim_{(\delta,\epsilon)\downarrow 0} \frac{P(x \le X \le x + \delta | y \le Y \le y + \epsilon)}{\delta} \\ &= \lim_{(\delta,\epsilon)\downarrow 0} \frac{P(x \le X \le x + \delta, y \le Y \le y + \epsilon)}{\delta \cdot P(y \le Y \le y + \epsilon)} \\ &= \lim_{(\delta,\epsilon)\downarrow 0} \frac{P(x \le X \le x + \delta, y \le Y \le y + \epsilon)}{\delta \epsilon} \cdot \frac{\epsilon}{P(y \le Y \le y + \epsilon)} \\ &= f(x,y) \cdot \frac{1}{h(y)} = \frac{f(x,y)}{h(y)}, \quad \text{provided } h(y) > 0 \end{split}$$

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Continuous = Density

Example - from before

Let X and Y be continuous RV's with joint probability density f

$$f(x,y) = \left\{ egin{array}{cc} 10\,x\,y^2, & , 0 < x < y \leq 1 \ 0, & ext{elsewhere} \end{array}
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Calculate conditional density of X, given $Y = \frac{1}{2}$ and calculate $P(X > \frac{1}{3}|Y = \frac{1}{2})$

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