Lecture 6: Probabilities and distributions (Part 4), Statistical Independence of RV

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Statistics (MAT1003)

April 24, 2012

Outline

1. Conditional Probabilities
2. Conditional Probability Distributions
   - Discrete
   - Continuous = Density
3. Statistical Independence of RVs

And now . . .

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   - Discrete
   - Continuous = Density
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book: Section 2.6

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$B|A$: The event B given $A = \{2\} = A \cap B$
But given that A occurs the sample space is actually reduced to $A = \{1, 2\}$
$P(B|A) = \frac{P(\{2\})}{P(\{1,2\})} = \frac{\frac{2}{10}}{\frac{3}{10}} = \frac{2}{3}$ (conditional probability)

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for any two events A and B, as long as $P(A) > 0$.

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Next: Conditional Probability Distributions (book: Section 3.4)

And now . . .

Conditional Probability Distribution

Let X and Y be discrete RVs with joint probability distribution f. The conditional probability of X given that Y has the value y is

$$f(x|y) = P(X = x|Y = y) = \frac{P(X=x, Y=y)}{P(Y=y)} = \frac{f(x,y)}{h(y)}$$

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Throwing 2 dice, X : max, Y : min.

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What is conditional probability distribution of X given $Y = 2$?
(Check at home) $P(Y = 2) = \frac{1}{4}$,
$x = 3, 4, 5, 6 : f(x, y) = \frac{1}{18}$; $x = 2 : f(x, y) = \frac{1}{36}$

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x	2	3, 4, 5, 6
$f(x, 2) = P(X = x\|Y = 2)$	$\frac{\frac{1}{36}}{\frac{1}{4}} = \frac{1}{9}$	$\frac{\frac{1}{18}}{\frac{1}{4}} = \frac{2}{9}$

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&= \lim_{(\delta,\epsilon)\downarrow(0,0)} \frac{P(x \le X \le x+\delta, y \le Y \le y+\epsilon)}{\delta \cdot P(y \le Y \le y+\epsilon)} \\
&= \lim_{(\delta,\epsilon)\downarrow(0,0)} \frac{P(x \le X \le x+\delta, y \le Y \le y+\epsilon)}{\delta\epsilon} \\
&\cdot \frac{\epsilon}{P(y \le Y \le y+\epsilon)} \\
&= f(x,y) \cdot \frac{1}{h(y)} = \frac{f(x,y)}{h(y)}, \quad \text{provided } h(y) > 0
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Same formula as for discrete case!

Example - from before

Let X and Y be continuous RV's with joint probability density f

$$f(x,y) = \begin{cases} 10\,x\,y^2, & 0 < x < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Calculate conditional density of X, given $Y = \frac{1}{2}$ and calculate $P(X > \frac{1}{3} | Y = \frac{1}{2})$

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(Check at home) $g(x) = \frac{10}{3}x - \frac{10}{3}x^4$, $0 < x < 1$ (0 elsewhere)
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$f(x|y) = \frac{10\,x\,y^2}{5\,y^4} = \frac{2x}{y^2}$, $0 < x < y < 1$ (0 elsewhere)

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$f(x|\frac{1}{2}) = P(X = x, Y = \frac{1}{2}) = \left[\frac{2x}{y^2}\right]_{y=\frac{1}{2}} = 8\,x$, $0 < x < \frac{1}{2}$ (0 elsewhere)

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$P(X > \frac{1}{3} | Y = \frac{1}{2}) = \int_{\frac{1}{3}}^{\frac{1}{2}} 8\,x \mathrm{d}x = \left[4\,x^2\right]_{x=\frac{1}{3}}^{\frac{1}{2}} = \frac{5}{9}$ (0 elsewhere)

And now . . .

Definition of statistical independence

Let X and Y be RV's with joint PDF f and let g and h be the marginal distributions of X and Y, respectively. Then X and Y are called independent if

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RVs: X : $\sharp$ reds from bag 1, Y : $\sharp$ reds from bag 2
Are X and Y dependent?

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$$f(x, y) = g(x) \cdot h(y) = \frac{\binom{3}{x}\binom{2}{2-x}\binom{4}{y}\binom{5}{4-y}}{\binom{5}{2}\binom{9}{4}}, x = 0, 1, 2,$$

$y = 0, 1, 2, 3, 4$